Oral Qualifying Exam Syllabus
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1 Combinatorics

Basic Enumeration: counting arguments, generating functions, binomial coefficients, recurrence relations, inclusion-exclusion principle, Stirling's formula

Set Systems: LYM inequality, Sperner's theorem, Erdős-Ko-Rado, Kruskal-Katona, Frankl-Wilson, Harper, Fisher's inequalitiy, projective geometries, Baranyai

Lattices and Posets: Dilworth, distributive lattices, Birkhoff representation theorem, Dilworth's theorem on dimension of distributive lattices, geometric lattices, Möbius inversion, Weisner, Dowling-Wilson

Correlation Inequalities: Harris-Kleitman, FKG, Ahlswede-Daykin , XYZ Theorem

Ramsey Theory: Ramsey's theorem, infinite Ramsey, König's lemma, probabilistic lower bounds, van der Waerden, Hales-Jewett, statement of Szemerédi's theorem

Discrepancy: Beck-Fiala

2 Graph Theory

Matching: König's Theorem, Hall's Theorem, Tutte's 1-factor theorem, stable matchings

Connectivity: greedy algorithm for minimum weight spanning tree, structure of 2-connected graphs, Menger's Theorem, Max-Flow-Min-Cut Theorem

Planarity: Euler's formula, Kuratowski's theorem, Wagner's Theorem

Coloring: 5 color theorem, Brooks' Theorem, Vizing's Theorem, Thomassen's 5-list-coloring of planar graphs, Hadwiger's conjecture, Galvin's Theorem, perfect graphs, Lovász's proof of the Weak Perfect Graph Theorem, statement of the Strong Perfect Graph Theorem

Extremal Problems: Turán's Theorem, the regularity lemma, Erdős-Stone, Chvátal-Rödl-Szemerédi-Trotter

3 Probabilistic Methods

Basics: linearity of expectation, alterations, Bonferroni inequalities, Chebyschev's inequality, Chernoff bound, alterations

Second Moment Method: general procedure, application to threshold function for having a certain graph as a subgraph

Lovász Local Lemma: symmetric and general versions, applications to hypergraph discrepancy, application to Ramsey lower bounds, and Latin transversals

Martingales: Azuma's inequality, edge and vertex exposure, application to chromatic number

Poisson Paradigm: Janson inequalities, application to number of triangles in $G_{n,p}$, Brun's sieve, application to number of isolated vertices in $G_{n,p}$

Random Graphs: monotone properties, existence of threshold functions, Bollobás-Thomason, probabilistic refutation of Hajós conjecture

Entropy: basic properties, Shearer's lemma, application to Minc conjecture

4 Probability Theory

Probability Spaces and Random Variables: algebras and σ-algebras, probability spaces, monotone class theorem, independence and product spaces, random variables, distribution functions, expectation, independence of random variables, convergence concepts for random variables, Kolmogorov's 0-1 law

Large Number Laws: weak law of large numbers, Borel-Cantelli lemma, strong law of large numbers, Kolmogorov's three series theorem

Stationary Processes: stationarity, measure-preserving transformations, Birkhoff's ergodic theorem, ergodic theorem for stationary processes

Central Limit Theorem: De Moivre-Laplace theorem, weak convergence and convergence in distribution, characteristic functions, continuity theorem, Lindeberg-Feller

Martingales: conditional expectation, definition of (sub)(super) martingales, stopping times, optional stopping theorems, application to random walks, Doob's martingale inequalities, Doob's upcrossing inequality, uniform integrability, martingale convergence theorems