Syllabus for the Oral Qualification Exam

Bin Guo

Committee: Sagun Chanillo, Zheng-Chao Han, Xiaojun Huang, Jian Song (chair)

I. Complex Geometry

1. Complex Manifolds
   - Stokes' Theorem
   - De Rham and Dolbeault Cohomology
   - De Rham Theorem and Dolbeault Theorem
   - Submanifolds and Subvarieties

2. Kähler Manifolds
   - Hodge Decomposition for Kähler Manifolds
   - Kodaira-Serre Duality
   - $\partial\bar{\partial}$-Lemma on Kähler Manifolds

3. Kodaira Embedding Theorem
   - Kodaira Vanishing Theorem
   - Curvature of Line Bundles
   - Adjunction Formulas
   - Chern Classes of Line Bundles
   - Blowing Up

4. Riemann Surfaces and Algebraic Curves
   - Embedding Riemann Surfaces
   - Riemann-Hurwitz Formula
   - The Genus Formula
   - The Riemann-Roch Formula

II. Elliptic Partial Differential Equations

1. Classical Solutions
   - Maximum Principles
   - Schauder Interior Estimates
   - Boundary and Global Estimates

2. Sobolev Spaces
   - Weak Derivatives
   - The $W^{k,p}$ Spaces
   - Sobolev Imbedding Theorems
   - The Morrey and John-Nirenberg Estimates

3. Weak Solutions
   - The Marcinkiewicz Interpolation Theorem
   - The Calderon-Zygmund Inequality
   - L^p Estimates
   - De Giorgi-Nash-Moser Iteration
   - Hölder and Harnack Estimates

III. Additional Topics

1. Evans-Krylov Theorem for Monge-Ampere Equations

2. Poincare-Lelong Formula

References

[1]   P. Griffiths and J. Harris, *Principles of Algebraic Geometry.*

[2]   D. Gilbarg and N. Trudinger, *Elliptic Partial Differential Equations of Second Order.*

[3]   M. Giaquinta, Multiple Integrals in the Calculus of Variations and Nonlinear Elliptic Systems.