Oral Qualifying Exam Syllabus

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Enumerative Combinatorics

1. **Basic Enumeration:** counting arguments, recurrence relations, inclusion-exclusion, pigeonhole principle, Stirling numbers, Bell numbers, Catalan numbers, Eulerian numbers, Fibonacci numbers

2. **Generating Functions:** formal power series, ordinary generating functions, exponential generating functions, Dirichlet series, fundamental theorem of exponential generating functions, Lagrange inversion

3. **Partially Ordered Sets:** chains/anti-chains, graded/ranked posets, Hasse diagrams, lattices, distributed lattices, geometric lattices, Birkhoff Representation Theorem, Dilworth's Theorem, Möbius inversion, Weisner's Theorem

4. **Discrete Probability:** basic discrete probability (mean, variance, moments), probability generating functions, Penney-Ante games

5. **Experimental Math and Applications:** applications of Goulden-Jackson Cluster Method (such as counting words of fixed length that avoid a set of bad subwords), Standard Young Tableaux (and enumerating them), plane and solid partitions, generating functions enumerating plane and solid partitions, bond and site percolation of a graph, computing percolation probabilities in graphs and matrices

References:
Graham, Knuth, and Patashnik, *Concrete Mathematics*
Noonan and Zeilberger, *The Goulden-Jackson Cluster Method: Extensions, Applications, and Implementations*
Stanley, *Enumerative Combinatorics, Vol. 1*

Wilf, *Generatingfunctionology*
Zeilberger, *Enumerative and Algebraic Combinatorics*

Graph Theory

1. **Basic graph theory:** basic graph definitions (trees, bipartite graphs, paths and cycles)
2. **Matching:** Hall's Theorem, König's Theorem, Tutte's Theorem
3. **Connectivity:** Menger's Theorem, Max Flow/Min Cut Theorem
4. **Planarity:** Euler's theorem, Kuratowski's theorem, Wagner's theorem
5. **Hamiltonicity:** Dirac's Theorem, Ore's Theorem, Bondy-Chvátal Theorem
6. **Coloring:** chromatic and edge-chromatic numbers, Brook's Theorem, Vizing's Theorem, chromatic polynomials, Perfect Graph Theorem
7. **Extremal:** Turán's Theorem, Erdős-Stone Theorem, statement and applications of Szemerédi's Regularity Lemma

References:
Diestel, *Graph Theory*

Hypergeometric Functions

1. **Definitions:** basic definition of hypergeometric series for single variable and multivariable, definition in terms of differential equations for single variable
2. **Summing:** formulas for the sum of a hypergeometric function when $x = 1$ (Explain using Euler integrals, combinatorics, and WZ theory)
3. **Other:** q-analogues, difference analogues, A-systems and connections with geometry

References:
Notes from Retakh's course *MATH 640:509 - Hypergeometric Functions*