Brent Young

Oral Qual Exam Syllabus

Major Topic: Mathematical Physics

I. Classical Physics

A. Newtonian Point Mechanics

i) Newton's Equation of Motion

a) Galilei Group

b) The Two Body Problem for attractive/repulsive $\frac{1}{r}$ potential (Part I)

c) The Vlasov-Poisson System

d) The Lorentz Force Law (point particle in uniform electric and magnetic field)

ii) Lagrangian Dynamics

a) Euler-Lagrange Equations

b) Equivalence to Newton's Equation of Motion

iii) Hamiltonian Dynamics

a) Legendre Transform

b) Equivalence to Newton's Equation of Motion

c) Phase Space as a Symplectic Manifold

d) Liouville's Theorem

e) The Two Body Problem for attractive/repulsive $\frac{1}{r}$ potential (Part II)

iv) Hamilton-Jacobi Formalism

a) Equivalence to Newton's Equation of Motion

b) Canonical Transformations

c) The Two Body Problem for attractive/repulsive $\frac{1}{r}$ potential (Part III)

B. Einsteinian Point Mechanics

i) Poincaré Group

ii) 4 Vectors and Invariants (proper time, wave operator, etc.)

C. Maxwell's Electromagnetic Field Theory

i) Derivation of Wave Equation in Lorenz Gauge
ii) Gauge Transformations
iii) Covariant form of Maxwell's Equations

II. **Quantum Physics**

A. Bohmian Mechanics

i) Basic equations (Schrödinger and guiding)
ii) Self-adjoint operators and unitary dynamics
iii) Heisenberg's Inequality (Uncertainty Principle)

B. Stability of matter

i) Sobolev Inequality applied to the Hydrogen Atom
ii) Many body problem (reduction to many Hydrogen problems)

III. **Classical PDEs of Mathematical Physics**

A. Laplace's Equation

i) Fundamental Solutions (including derivations)
ii) Green's Identities
iii) Mean Value Theorem
iv) Green's Functions and the Poisson Kernel
v) Dirichlet Problem on the Unit Ball
vi) Uniqueness of Dirichlet Problem Solutions via the Maximum Principle
vii) Perron's Existence Method for the Dirichlet Problem

B. The Wave Equation

i) C^2 Uniqueness via Energy
ii) Fundamental Solutions (derivation for n = 1,2, and 3)
iii) Duhamel's Principle

Minor Topic: Analysis

I. Hilbert Spaces

A. Closed subspace decomposition

B. Self duality

C. Bessel's Inequality

D. Completeness

II. Elements of Fourier Analysis

A. Schwarz Space as a Fréchet Space

B. Convolutions

- i) Young's Inequality
- ii) Properties

C. Approximations of the identity

- i) Density of C_c^∞ in L^p
- ii) C^∞ Urysohn Lemma

D. Fourier Transform on $\mathbb{R}^n$

- i) Properties
- ii) Riemann-Lebesgue Lemma
- iii) Fourier Inversion
- iv) Plancherel Theorem

III. The Theory of Distributions

A. Definitions of convergence of test functions and continuity of functionals

B. Derivatives and convolutions of distributions/test functions

C. Density of $C_c^\infty(U)$ in $D'(U)$

D. Tempered Distributions

- i) Examples
- ii) Fourier Transform

IV. L^2 Sobolev Spaces

A. Definition of H_s

B. Equivalent norms

C. Properties (including density of H_s in H_t for $t < s$)

D. Sobolev Embedding Theorem for H_s

E. Rellich's Compactness Theorem

F. Definition of Localized H_s Spaces

G. The Elliptic Regularity Theorem for constant coefficients