

**Oral Qualifying Exam Syllabus**  
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Week of Tuesday, January 17, 2006(tentative)

Committee (in alphabetical order): J. Beck, H. Iwaniec, J. Tunnell, C. Weibel.

1. Modular Forms

- (a) Modular Forms for the full modular group and its congruence subgroups
- (b) Eisenstein series
- (c) Structure of the ring of modular forms
- (d) Hecke operators
- (e) Automorphic  $L$ -series

2. Elliptic Curves

- (a) Elliptic functions and the  $j$ -invariant
- (b) Elliptic curves over the complex field
- (c) Elliptic curves over finite fields, Hasse-Weil Theorem
- (d) Elliptic curves over local fields
- (e) Elliptic curves over the rational numbers, Mordell-Weil Theorem, and descent on elliptic curves
- (f) Hasse-Weil  $L$ -functions of elliptic curves

3. Analytic Number Theory

- (a) Analytic properties of Dirichlet  $L$ -functions and the Riemann zeta function
- (b) Primes in arithmetic progressions
- (c) Prime number theorem and prime number theorem for arithmetic progressions
- (d) Siegel zero problem

4. Algebraic Number Theory

- (a) Invariants of number fields: rings of integers, different, and discriminant
- (b) Dedekind domains
- (c) Arithmetic of number fields: splitting and ramification of primes
- (d) Class groups, finiteness of class number
- (e) Dirichlet's Unit Theorem

## References

- [TA1] T. Apostol, Introduction to Analytic Number Theory, Springer-Verlag, 1976.
- [TA2] T. Apostol, Modular Functions and Dirichlet Series in Number Theory, Springer-Verlag, 1976.
- [HI] H. Iwaniec, prime numbers course notes, Fall, 2005.
- [SL] S. Lang, Algebraic Number Theory, Springer-Verlag, 1986.
- [TM] T. Miyake, Modular Forms, Springer-Verlag, 1989. (selected topics)
- [JS] J. Silverman, The Arithmetic of Elliptic Curves, Springer-Verlag, 1986. (basic parts)
- [JT1] J. Tunnell, elliptic curves course notes, Fall, 2004.
- [JT2] J. Tunnell, modular forms course notes, Spring, 2005.