Topics for oral qualifying exam for Bud Coulson
Fall, 2012

Major topic: Vertex operator algebras

1. Definitions and properties.
   (a) Formal calculus.
   (b) The notions of vertex algebra and of vertex operator algebra, and basic properties.
   (c) Rationality, commutativity and associativity; equivalence of various formulations, including "weak" formulations.

2. Representations of vertex (operator) algebras.
   (a) The notion of module and basic properties.
   (b) Weak vertex operators.
   (c) The structure of the canonical weak vertex algebra. Local subalgebras and vertex subalgebras of the canonical weak vertex algebra.
   (d) The equivalence between modules and representations.
   (e) General construction theorems for vertex (operator) algebras and modules.

3. Examples of vertex (operator) algebras and modules.
   (a) Vertex (operator) algebras and modules based on the Virasoro algebra.
   (b) Vertex (operator) algebras and modules based on affine Lie algebras.
   (c) Vertex (operator) algebras and modules based on Heisenberg Lie algebras.
   (d) Vertex (operator) algebras and modules on even lattices.
   (e) Vertex operator construction of the affine Lie algebras corresponding to A_n, D_n and E_n.

4. Affine Lie algebras
   (a) Classification of affine Lie algebras, twisted and untwisted.
   (b) Explicit realization of affine Lie algebras.
   (c) Affine root systems and Weyl groups.
5. The fundamentals of logarithmic tensor product theory for generalized modules for a conformal vertex algebra.
   (a) The notions of conformal and Möbius vertex algebra.
   (b) The notions of module and of generalized module.
   (c) Opposite vertex operators, and contragredient modules and generalized modules.
   (d) The notions of intertwining operator, logarithmic intertwining operator and fusion rule; basic properties.
   (e) Logarithmic formal calculus.
   (f) The notion of $P(z)$-tensor product of generalized modules.

Minor topic: Descriptive Set Theory

1. Polish and Standard Borel Spaces
   (a) Basic definitions and examples.
   (b) The Borel Isomorphism Theorem.
   (c) Borel-generated topologies and the Ramsey-Mackey Theorem.
   (d) Sequential trees. Systems of sets and their associated maps. Souslin, Lusin and Cantor schemes. The Souslin operation $\mathcal{A}$.

2. The Borel and Projective Hierarchies

   (a) Basic definitions, facts including closure properties.
   (b) Existence of universal sets for each. Non-collapsing of each.
   (c) Every uncountable Polish space contains an analytic set that is not Borel.
   (d) Equivalence of various definitions of analytic sets.
   (e) Every coanalytic set is a union of $\aleph_1$ Borel sets.
   (f) Definitions of Σ^1_1-complete, Π^1_1-complete. WF is Π^1_1-complete.
   (g) Regularity properties: Every analytic subset of a Polish space is measurable, has the Baire property, and has the perfect set property.
   (h) Souslin's Theorem and the First Separation Theorem for analytic sets.

References

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[H] J. Humphreys, Introduction to Lie Algebras and Representation Theory, Second Printing, Revised, Springer-Verlag, 1972.

[LL] J. Lepowsky and H. Li, Introduction to Vertex Operator Algebras and Their Representations, Progress in Math., Vol. 227, Birkhäuser, Boston, 2003.

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[M] Y.N. Moschovakis, Descriptive Set Theory: Second Edition, Mathematical Surveys and Monographs, Volume 155, 2009

[S] S.M. Srivastava, A Course on Borel Sets, Springer-Verlag, 1998.