

Oral Exam Syllabus
Benjamin Bunting
June 24, 2005

Committee: Michael Vogelius, Stephen Greenfield, Zheng-Chao Han and Michael Kiessling

I. Partial Differential Equations

i. Laplace Equation

- a. Derivation of the fundamental solution
- b. Mean value formula and maximum principles
- c. Analyticity of harmonic functions via gradient estimates
- d. Existence Technique 1: The Perron method and Harnack's inequality
- e. Existence Technique 2: Discretization of the Laplace equation
- f. Dirichlet's Principle

ii. Heat Equation

- a. Derivation of the fundamental solution
- b. Maximum principles
- c. Energy methods

iii. Wave Equation

- a. Solution by spherical means
- b. Uniqueness and finite propagation speed via energy methods

iv. Sobolev Spaces

- a. Definitions: H^s , $W^{k,p}$
- b. Density of C^∞ in $W^{k,p}$
- c. Extensions, traces, compactness
- d. Sobolev inequalities
- e. Fourier transform methods

v. Second Order Elliptic Equations

- a. Definition of elliptic equations, weak solutions
- b. Existence of weak solutions via Lax-Milgram theorem, energy estimates, Fredholm alternatives

- c. Maximum principles weak and strong, Hopf lemma
- d. Regularity of solutions in H^k
- e. Existence via Schauder estimates in C^α
- f. Consequences of Riesz-Thorin
- g. Eigenvalues of symmetric elliptic operators

II. Functional Analysis

i. Hilbert Spaces

- a. Riesz representation and consequences
- b. Orthonormal bases
- c. Compact operators on a Hilbert space
- d. Diagonalization of compact self-adjoint operators
- e. Application: Sturm-Liouville systems

ii. Banach Spaces

- a. Linear functionals
- b. Hahn-Banach theorem
- c. Applications: Banach limits, Runge's theorem
- d. Baire category, open mapping, closed graph theorems
- e. The principle of uniform boundedness

iii. Weak Topologies

- a. Alaoglu's theorem, consequences of reflexivity
- b. Schauder, Markov-Kakutani, Ryll-Nardzewski fixed point theorems
- c. Application: Haar measure on a compact group

References

- i. Evans, Lawrence C., *Partial Differential Equations*
- ii. Folland, Gerald B., *Introduction to Partial Differential Equations. Second Edition*
- iii. Han, Qing and Lin, Fanghua, *Elliptic Partial Differential Equations*
- iv. Conway, John B., *A Course in Functional Analysis*
- v. Folland, Gerald B., *Real Analysis*