Oral Qualifying Examination Syllabus

Justin Bush, March 2009

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Topic 1: Algebraic Topology

- Fundamental Group
  - Van Kampen Theorem
  - Covering Spaces
- Simplicial and Singular Homology
  - Exact sequences
  - Axioms for homology
  - Homology and Fundamental Group
  - Simplicial approximation
  - Universal coefficients for homology
- Simplicial and Singular Cohomology
  - Universal coefficients for cohomomology
  - Exact sequences
  - Cup, cap, cross products
  - Künneth formula
  - Poincaré duality
  - Alexander duality
- CW complexes
  - Cellular homology and cohomology
  - Cellular approximation
  - CW approximation
- de Rham Cohomology

  - Mayer-Vietoris sequence
  - Poincaré duality
  - The de Rham Theorem
  - Thom isomorphism
- The h-cobordism Theorem
  - Morse functions
  - Cobordism category
  - Poincaré conjecture in high dimensions
- Geometric Topology
  - The Schoenfliess Theorem
  - Whitehead's Theorem
  - Hurewicz Theorem
  - Handle decompositions

References:

[1] Allen Hatcher, *Algebraic Topology*, Cambridge University Press, 2001.

[2] John Milnor, *Lectures on the h-cobordism Theorem*, Princeton University Press, 1965.

[3] Raoul Bott and Loring Tu, *Differential Forms in Algebraic Topology*, Springer, 1982.

[4] Steve Ferry, Geometric Topology notes.

Topic 2: Probability and Martingales

- Probability Theory
  - Conditional expectation
  - Markov and Chebyshev inequalities
  - Borel-Cantelli Lemmas
  - Weak and Strong Law of Large Numbers
  - Characteristic Functions

- Central Limit Theorem

- Martingales
  - Doob's upcrossing lemma
  - Martingale convergence theorems
  - Doob's inequality
  - Stopping times
  - Optional sampling

References:

[1] David Williams, *Probability With Martingales*, Cambridge University Press, 1991.

[2] Patrick Billingsley, *Probability and Measure*, Wiley-Interscience, 1995.