Topics for oral qualifying exam for Corina Calinescu Spring, 2003

Major topic: Vertex operator algebras

1. Definitions and properties.

    (a) Formal calculus.

    (b) The notions of vertex algebra and of vertex operator algebra, and basic properties.

    (c) Rationality, commutativity and associativity; equivalence of various formulations, including "weak" formulations.

2. Representations of vertex (operator) algebras.

    (a) The notion of module and basic properties.

    (b) Weak vertex operators.

    (c) The structure of the canonical weak vertex algebra. Local subalgebras and vertex subalgebras of the canonical weak vertex algebra.

    (d) The equivalence between modules and representations.

    (e) General construction theorems for vertex (operator) algebras and modules.

3. Examples of vertex (operator) algebras and modules.

    (a) Vertex (operator) algebras and modules based on the Virasoro algebra.

    (b) Vertex (operator) algebras and modules based on affine Lie algebras.

    (c) Vertex (operator) algebras and modules based on Heisenberg Lie algebras.

    (d) Vertex (operator) algebras and modules on even lattices.

    (e) Classification of the irreducible modules for the simple vertex operator algebras associated to standard modules.

4. The notions of intertwining operator and fusion rule, basic properties.

Minor topic: Kac-Moody algebras

1. Definitions and properties.

   (a) Triangular and root space decompositions.

   (b) The invariant bilinear form and the generalized Casimir element.

   (c) Integrable representations of Kac-Moody algebras and the Weyl group.

2. A classification of generalized Cartan matrices.

3. Real and imaginary roots.

   (a) Definitions and properties.

   (b) An explicit description of the root system of a Kac-Moody algebra.

4. Affine algebras.

   (a) The normalized invariant form, the root system, and the Weyl group.

   (b) Affine algebras as central extensions of loop algebras.

5. The representation theory of Kac-Moody algebras.

   (a) The category O. Highest-weight modules and Verma modules.

   (b) The irreducibility and complete reducibility in O.

   (c) Formal characters of modules from O.

   (d) Integrable highest-weight modules over Kac-Moody algebras. The character formula, the denominator formula and the multiplicity formula.

   (e) The denominator formula for $\widehat{sl_2(C)}$. The Jacobi triple product identity.

   (f) On the defining relations for a symmetrizable Kac-Moody algebra.

References

[FHL]   I. Frenkel, Y.-Z. Huang and J. Lepowsky, On Axiomatic Approaches to Vertex Operator Algebras and Modules, Memoirs Amer. Math. Soc. 104 (1993).

[FLM]   I. Frenkel, J. Lepowsky and A. Meurman, Vertex Operator Algebras and the Monster, Academic Press, 1988.

[K]   V. Kac, Infinite dimensional Lie algebras, third edition,Cambridge University Press, 1990.

[LL]   J. Lepowsky and H. Li, Introduction to vertex operator algebras and their representations, to appear.

[L]   H. Li, Local systems of vertex operators, vertex superalgebras and modules, J. Pure and Applied Algebra 109 (1996), 143-195.