

Oral Qualifying Exam Syllabus

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1. Partial Differential Equations

(a) Constant coefficient case: Laplace, Heat and Wave equations.

- Fundamental solutions.
- Mean-value formulas.
- Important properties.
- Harnack's Inequality.
- Green's functions.
- Energy methods.

(b) Sobolev spaces

- Completeness, reflexivity, density, extensions, traces.
- Important Inequalities: Sobolev-Gagliardo-Nirenberg, Morrey, Poincaré.
- Embeddings, Borderline cases.

(c) Second Order Elliptic equations.

- Weak Solutions.
- Existence: Lax-Milgram, Fredholm Alternative.
- Interior and Boundary Regularity.
- $W^{2,p}$ estimates: Calderon-Zygmund inequality.
- Maximum principle, Hopf Lemma.
- Eigenvalues and Eigenfunctions.

2. Functional Analysis

(a) Hilbert Spaces.

- Riesz representation.
- Compact operators.
- Spectral theory of self-adjoint compact operators.

(b) Banach Spaces.

- Linear functionals.
 - Hahn-Banach theorem.
 - Classical Theorems: Baire Category, Open Mapping, Closed Graph, Banach-Steinhaus.
 - Weak Topology, Reflexivity, Banach-Alaoglu.
 - Fixed point theorems.
- (c) Theory of Distributions.
- Definition, Derivative of a Distribution, Applications to ODE.
 - Convolution Equations, Heaviside's Calculus.
 - Density of $C_0^\infty(U)$ in $D'(U)$.
 - Classical Examples: Applications.
 - Tempered Distributions, Fourier Transform.
- (d) Degree Theory.
- Brouwer degree, Brouwer fixed point Theorem, Borsuk's Theorem.
 - Leray-Schauder degree, Leray-Schauder fixed point Theorem.
 - Applications.

References

- [1] H. Brezis. *Análisis funcional*. Alianza Editorial, Madrid, 1984.
- [2] L. C. Evans. *Partial Differential Equations*. (Graduate Studies in Mathematics, V. 19) GSM/19. American Mathematical Society, 1998.
- [3] I. Fonseca and W. Gangbo. *Degree Theory in Analysis and Applications*. Oxford Science Publications, 1995
- [4] D. Gilbarg and N. S. Trudinger. *Elliptic Partial Differential Equations of Second Order*. Springer-Verlag Berlin Heidelberg New York (1977) Reprint of the 1998 Edition.
- [5] Various Lecture Notes.