# ORAL QUAL SYLLABUS:
# SET THEORY; ERGODIC THEORY

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**Set Theory.**
- Basic set theory (Jech 1-9)
  - Königs lemma, cardinal arithmetic in ZFC
  - Fodor's lemma, Solovay's theorem
  - Ramsey's theorem
  - there exists an Aronsajn tree
  - Shanin's root system lemma ($\times 2$)
- Descriptive set theory (Jech 11)
  - elementary properties of Polish spaces
  - non-collapsing of the Borel hierarchy
- Models of set theory (Jech 12,13)
  - Mostowski's collapsing theorem
  - relativization and absolute formulas
  - reflection principle
  - $L$ is a model of ZFC, $L_\alpha$ is absolute (statements)
  - condensation lemma
  - $L$ is a model of GCH, $\diamondsuit$
- Forcing basics (Kunen, ch. VII, VIII)
  - fundamental forcing theorems (statements)
  - chain and closure conditions on a partial order
  - mixing and maximal lemmas
  - Cohen and Lévy forcing
  - product and iterated forcing
  - factor lemma (statement)
  - independence of MA+¬CH
  - consequences of MA, e.g. $2^\omega$ is regular
  - Easton supported forcing, Easton's theorem
- Large cardinals (Jech 17, Kanamori 5, Joel's notes)
  - measurable cardinals and embeddings $j : V \to M$
  - Lévy-Solovay theorem (small forcing and measurables)
  - lifting criterion; diagonalization criterion
  - find a weakly compact cardinal in $L$
  - supercompact cardinals have a Laver function
  - Silver's theorem (failure of GCH at a measurable)
  - Laver's theorem (the Laver preparation)
  - Reinhardt cardinals

**Ergodic Theory.**

- Real analysis (Walters, others)
  - regularity properties of Borel measures
  - Jensen's inequality
  - Riesz representation theorem (statement)
  - spectral theorem for unitary operators (statement)
  - conditional expectation
  - existence and properties of Haar measure (statements)
  - characters of compact abelian groups (statements)
- Measure-preserving transformations (Walters 1)
  - recurrence theorem
  - equivalent formulations of ergodicity
  - equivalent formulations of weak and strong mixing
  - Von-Neumann mean ergodic theorem
  - Garsia maximal inequality
  - Hopf ergodic theorem
  - Birkhoff ergodic theorem
- Examples
  - ergodic transformation on $\mathbb{R}$
  - group rotations, endomorphisms, and affine transformations
  - Bernoulli shifts
- Discrete and continuous spectrum (Walters 2,3)
  - notions of similarity
  - multiplication theorem
  - spectrum of an ergodic transformation
  - continuous spectrum theorem
  - discrete spectrum theorem
  - representation theorem for discrete spectrum
- Topological dynamics (Walters 5.2-4)
  - minimality
  - transitivity
- Unique ergodicity (Walters 6.5)
  - the topological space of invariant measures
  - existence of ergodic measures
  - characterizations of unique ergodicity
  - basic facts about $g$-measures

**References.**

- Jech, Thomas. Set Theory. *Third* Edition.
- Kanamori, Akihiro. The Higher Infinite.
- Kunen, Kenneth. Set Theory: An Introduction to Independence Proofs.

- Halmos, Paul R. Lectures on Ergodic Theory.
- Walters, Peter. An Introduction to Ergodic Theory.