Derek Hansen's Syllabus for the Oral Qualifying Exam

1. Functional Analysis

   (a) basics of Banach spaces

      i. operations on Banach spaces (direct sums and quotients)
      ii. linear functionals, the Hahn-Banach theorem (analytic and geometric forms), dual spaces
      iii. the Baire Category theorem and its consequences, namely, the Banach-Steinhaus theorem, the open and inverse mapping theorems, and the closed graph theorem
      iv. basics of Hilbert spaces (Riesz lemma, adjoints, orthonormal bases, etc.)
      v. weak and weak-$*$ topologies, the Banach-Alaoglu theorem, the Eberlein Šmulian theorem, Mazur's theorem

   (b) measure theory on compact Hausdorff spaces (the Riesz representation theorem)

   (c) bounded operator theory

      i. adjoints
      ii. resolvent set, resolvent operator and the spectrum
      iii. compact operators
      iv. the spectrum of a compact operator
      v. the Fredholm alternative
      vi. the spectral theorem for compact normal operators on a Hilbert space

2. Sobolev Spaces

   (a) $H^s(\mathbb{R}^n)$ defined via the Fourier transform
   (b) $W^{k,p}$ spaces
   (c) density of $C^\infty(\bar{\Omega})$ in $W^{k,p}(\Omega)$
   (d) global extension
   (e) the trace operator
   (f) the Gagliardo-Nirenberg-Sobolev inequality
   (g) Morrey's inequality
   (h) the Rellich-Kondrakov theorem
   (i) Lipschitz functions and $W^{1,\infty}$
   (j) Rademacher's theorem

3. Second Order Elliptic PDE

   (a) Laplace's Equation

      i. the fundamental solution

   ii. solving Poisson's equation, $\Delta u = f$ in Ω, where Ω is a bounded domain and f is locally Hölder continuous, with the Newtonian potential of f
   iii. mean value formula and maximum principle for subharmonic functions
   iv. basic Harnack's inequality
   v. Green's function
   vi. basic properties of harmonic functions (analyticity, a uniformly bounded family of harmonic functions has a u.c.c. convergent subsequence)
   vii. Perron's method of subharmonic functions
   viii. existence of solutions to Dirichlet and Neumann problems via layer potentials

(b) weak and strong maximum principles, uniqueness theorems for boundary value problems

(c) definition of weak solution

(d) existence via energy estimates and the Lax-Milgram theorem

(e) existence via the Fredholm alternative

(f) eigenvalues and eigenfunctions, the Rayleigh-Ritz characterization of the principle eigenvalue of a symmetric operator

(g) interior and boundary regularity of solutions

References:

1. Gilbarg, David and Trudinger, Neil S. *Elliptic Partial Differential Equations of Second Order*

2. Evans, Lawrence. *Partial Differential Equations*

3. Folland, Gerald B. *Introduction to Partial Differential Equations*

4. Brézis, Haïm. *Analyse Fonctionelle*

5. Reed, Michael and Simon, Barry. *Functional Analysis*

6. Protter, Murray H. and Weinberger, Hans F. *Maximum Principles in Differential Equations*