Oral Qualifying Exam Syllabus

Goran Djanković

April 2006

Committee (in alphabetical order): H. Iwaniec, S. Miller, J.Tunnell, C. Weibel

1. **Automorphic Forms and Spectral Theory**

   (a) The hyperbolic plane as a homogeneous space and the classification of motions

   (b) Fuchsian groups and fundamental domains

   (c) Congruence subgroups

   (d) The double coset decomposition; Kloosterman sums

   (e) Laplace and Maass operators

   (f) Automorphic forms- holomorphic and Maass;
   Fourier expansions of automorphic forms

   (g) Eisenstein and Poincaré series (holomorphic and spectral)

   (h) Hilbert space of holomorphic cusp forms; $\mathcal{C}(\Gamma\backslash\mathbb{H}) = \mathcal{L}^2(\Gamma\backslash\mathbb{H}) \ominus \mathcal{E}(\Gamma\backslash\mathbb{H})$

   (i) Petersson's formula

   (j) The Riemann-Roch theorem and the dimension of the linear space of
   holomorphic modular forms

   (k) Hecke operators; Atkin-Lehner theory

   (l) Automorphic L-functions - classical $GL(2)$-theory

   (m) Automorphic Green function

   (n) Analytic continuation and functional equations for the Eisenstein series

   (o) The spectral theorem

   (p) Selberg's trace formula

2. **Analytic Number Theory**

   (a) Primes in arithmetic progressions-Dirichlet's theorem

   (b) The Riemann zeta function

   (c) Dirichlet $L-$functions

   (d) Counting zeros of $L-$functions

   (e) The zero-free region; Landau-Siegel theorem

   (f) Explicit formulas

   (g) Prime number theorem and PNT for arithmetic progressions

   (h) Bilinear forms and the large sieve inequalities

   (i) Bombieri-A.I. Vinogradov theorem

3. **Algebraic Number Theory**

   (a) Lattices and orders in number fields; discriminant of a lattice in a number field

   (b) Minkowski's lattice point theorem; Minkowski's bound; Hermite's theorem

   (c) Class group

   (d) Dirichlet's unit theorem

   (e) The different and ramification

   (f) Splitting of primes in Galois extensions

4. **Elliptic Curves**

   (a) Elliptic curves- group law and isogenies;

   (b) Elliptic functions and elliptic curves over $\mathbb{C}$

   (c) Elliptic curves over finite fields; Hasse's theorem

   (d) Hasse-Weil L-functions of elliptic curves