

Oral Syllabus of David Nacin

Minor Topic: The Automorphism Tower Problem
Simon Thomas: The Automorphism Tower Problem

1. The Automorphism Tower Problem
 - (a) The Automorphism Tower Problem
 - (b) Some Fundamental Results
 - (c) Some Examples of Automorphism Towers
 - (d) The Infinite Dihedral Group
2. Wielandt's Theorem
 - (a) Automorphism Towers of Finite Groups
 - (b) Subnormal Subgroups
 - (c) Finite p-groups
3. Some Examples of Automorphism Towers
 - (a) The Automorphism Tower Theorem
 - (b) T_k is increasing
 - (c) A Better Bound
 - (d) The Automorphism Tower Problem Revisited
4. The Normalizer Tower Technique
 - (a) Normalizer Towers
 - (b) Coding Structures in Graphs
 - (c) Coding Graphs in Fields
 - (d) A Technical Lemma
5. Hamkins' Theorem
 - (a) Automorphism Towers of Arbitrary Groups
 - (b) Some Examples

Major Topic: Lie Algebras and Representations
James Humphreys: Introduction to Lie Algebras and Representation Theory

1. Definitions and First Examples Definitions and Classical Algebras, Lie Algebras of Derivations, Structure Constants and Abstract Lie Algebras

2. Ideals and Homomorphisms, Important Ideals, Homomorphisms and Representations, Automorphisms
3. Solvable and Nilpotent Lie Algebras, Solvability and Nilpotency, Engel's Theorem
4. Theorems of Lie and Cartan, Lie's Theorem, Jordan-Chevalley Decomposition, Cartan's Criterion
5. Killing Form, Criterion for Semisimplicity, Inner Derivations, Abstract Jordan Decomposition
6. Complete Reducibility of Representations, Modules, Casimir Element of a Representation, Weyl's Theorem, Preservation of Jordan Decomposition
7. Representations of $\mathfrak{sl}(2, \mathbb{F})$, Weights and Maximal Vectors, Classification of Irreducible Modules
8. Root Space Decomposition, Maximal Toral Subalgebras and Roots, Centralizer of \mathfrak{h} , Orthogonality Properties, Integrality Properties, Rationality Properties
9. Axiomatics, Reflections in a euclidean space, root systems, examples, pairs of roots
10. Simple Roots and Weyl group, Bases and Weyl chambers, Lemmas on simple roots, The Weyl group, Irreducible root systems
11. Classification, Cartan matrix of Φ , Coxeter graphs and Dynkin diagrams, Irreducible components, Classification theorem
12. Construction of root systems and automorphisms, Construction of types A-G, Automorphisms of Φ
13. Abstract theory of weights, Weights, Dominant Weights, The Weight δ , Saturated sets of weights
18. Generators and Relations, Relations Satisfied by L and Consequences, Serre's Theorem, Application: Existence and Uniqueness
20. Weights and Maximal Vectors, Weight Spaces, Standard Cyclic Modules, Existence and Uniqueness
21. Finite Dimensional Modules, Necessary Condition, Sufficient Condition, Weight Strings and Weight Diagrams, Generators and Relations for $V(\lambda)$
22. Multiplicity Formula, A Universal Casimir Element, Traces on Weight Spaces, Freudenthal's Formula, Examples, Formal Characters
23. Characters, Invariant Polynomial Functions, Standard Cyclic Modules and Characters, Harish-Chandra's Theorem

24. Formulas of Weyl, Kostant, and Steinberg, Some Functions on H^* , Kostant's Multiplicity Formula, Weyl's Formulas, Steinberg's Formula

James Lepowsky: Lectures on Kac-Moody Lie Algebras

1. Kac-Moody Lie Algebras
2. The Weyl Group
3. Standard Modules
4. The Numerator Formula
5. The Symmetrizable Case and the Invariant Form
6. The Casimir Operator and Consequences
7. The Weyl-Macdonald-Kac Formulas

Nathan Jacobson: Lie Algebras

- Complete Reducibility of the Representations of Semisimple Algebras
- Cohomology Groups of a Lie Algebra
- Definition and Basic Properties of Universal Enveloping Algebras
- The Poincare-Birkhoff-Witt Theorem
- Filtration and Graded Algebras
- Free Lie Algebras
- Cohomology of Lie Algebras

Igor Frankel/James Lepowsky/Arne Meurman: Vertex Operator Algebras and the Monster

1. Lie Algebras
 - 1.1 Algebras
 - 1.2 Modules
 - 1.3 Algebra Constructions
 - 1.4 Module Constructions
 - 1.5 Induced Modules
 - 1.6 Affine Lie Algebras
 - 1.7 Heisenberg Algebras
 - 1.8 Contravariant Forms
 - 1.9 The Virasoro Algebra
 - 1.10 Graded Dimension

- 2. Formal Calculus
 - 2.1 Formal Series
 - 2.2 Derivations
 - 2.3 Affine Lie Algebras Via Formal Variables
- 3. Realizations of $sl(2)^\wedge$ by Twisted Vertex Operators
 - 3.1 The Affine Lie Algebra $sl(2)^\wedge$
 - 3.2 The Twisted Vertex Operators $X_{Z+1/2}(\alpha, z)$
 - 3.3 Normal Ordering
 - 3.4 Some Commutators
- 4. Realizations of $sl(2)^\wedge$ by Untwisted Vertex Operators
 - 4.1 2-Cocycles
 - 4.2 Normal Ordering
 - 4.3 Some Commutators
 - 4.4 Irreducible Representations of $sl(2)^\wedge$ and $sl(2)^\wedge[\theta_1]$
- 5. Central Extensions
 - 5.1 2-Cocycles
 - 5.2 Commutator Maps
 - 5.3 Extraspecial 2-Groups
 - 5.4 Automorphisms of Central Extensions
 - 5.5 Representations of Central Extensions
- 6. The Simple Lie Algebras A_n, D_n, E_n
 - 6.1 Lattices
 - 6.2 A Class of Lie Algebras
 - 6.3 The Cases A_n, D_n, E_n
 - 6.4 A Group of Automorphisms of \mathfrak{g}