Topics for oral qualifying exam for David Radnell, June, 2000

Major topic: Geometric theory of Vertex operator algebras

The general theory of vertex operator algebras, simple examples, the basic geometry of vertex operator algebras, determinant lines over Riemann surfaces with parametrized boundaries and the diffeomorphism group of S^1 as presented in [FLM] (Chapters 1–8), [FHL], [H] and [L]. The specific topics are:

The notions of vertex algebra and of vertex operator algebra, and basic properties

Rationality, commutativity and associativity (various formulations, including "weak" formulations, as presented in [FLM], [FHL] and [L])

Examples (using local Systems theory): Vertex algebras and vertex operator algebras based on Heisenberg Lie algebras, on the Virasoro algebra and on even lattices.

The moduli spaces of spheres with tubes, the sewing operation and sewing equation, the geometric and formal solutions of the sewing equation, examples for the sewing operation

The geometric interpretations of vertex operators and vacua, the geometric meanings of commutativity, associativity, skew-symmetry, $L(-1)$-derivative property, vacuum property and creation property, as presented

The notion of geometric vertex operator algebra and the isomorphism theorem

Determinant lines of Fredholm operators, basic categorical properties, basic canonical isomorphisms

Determinant lines over Riemann surfaces with parametrized boundaries, the construction of the sewing canonical isomorphisms, the associativity of the sewing canonical isomorphisms

The groups $GL_{\mathrm{res}}(H)$ and $\mathrm{Diff}S^1$, the central extension of $GL_{\mathrm{res}}(H)$, two constructions of the basic central extensions of $\mathrm{Diff}S^1$

Minor topic: Functional Analysis

Basic Hilbert and Banach space theory. Banach algebras and elementary spectral theory. Ideals of bounded operators. Fredholm operators.

Hilbert and Banach Spaces: [R],[S],[D]
: Existence of orthogonal bases. Dual space and adjoint operators. Riesz representation theorem. $C([a,b])$ and $C([a,b])^*$. Topologies (weak, weak* and operator). Banach-Alaoglu, Hahn-Banach and Open Mapping theorems. Embedding into $C(X)$.

Banach algebras and spectral theory [D],[M],[R]
: Stone-Weierstrass Theorem. Example (Disc Algebra). Elementary properties of the spectrum and spectral radius. Spectral mapping theorem. Gelfand Representation for abelian Banach Algebras.

Operator Ideals: [M],[BS]
: Definition of, and relation between, finite rank, compact, Hilbert-Schmidt and trace class operators. Example using kernels. Correspondence between operator ideals and certain sequence spaces (Calkin Theory).

Fredholm operators: [D],[M]
: Fredholm Alternative. Relation to existence/uniqueness of differential equations. Atkinson Theorem.

References

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[FLM] I. Frenkel, J. Lepowsky and A. Meurman, Vertex Operator Algebras and the Monster, Academic Press, 1988.

[H] Y.-Z. Huang, Two-dimensional conformal geometry and vertex operator algebras, Progress in Math., Vol. 148, Birkhäuser Boston, 1997.

[L] H. Li, Local systems of vertex operators, vertex superalgebras and modules, J. Pure and Applied Algebra 109 (1996), 143-195.

[D] R. G. Douglas, Banach Algebra Techniques in Operator Theory (second edition), Graduate Texts in Mathematics 179, Springer, 1998

[M] G. J. Murphy, C^*-Algebras and Operator Theory, Academic Press, 1990

[R] W. Rudin, Functional Analysis, McGraw Hill, 1973

[S] G. F. Simmons, Topology and Modern Analysis, McGraw Hill, 1963

[BS] B. Simon, Trace Ideals and their Application, Cambridge university Press, 1979