Oral Qualifying Exam Syllabus

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Major Topic: Noncommutative Algebra

1. General noncommutative ring theory ([Lam] §§1-4, 7-9)
   - Semi-simple modules and rings
   - Wedderburn-Artin Theory
   - Jacobson Radical
   - modules over kG/representations of finite groups, characters
   - linear groups: Burnside's Theorem
2. Quasideterminants ([GGRW])
   - definition - in terms of inverses and recursive
   - properties - e.g. row/column relations, Sylvester's theorem
   - applications - e.g. Vandermonde quasideterminant, Vieta theorem, symmetric functions, quasi-Plucker coordinates
3. Algebras related to roots of equations
   - Q_n, $A(\Gamma)$ - definition, describe linear basis[GRSW]
   - Bergmans Diamond Lemma ([Bergman] §§1-3)
   - factorization of twisted polynomial rings: remainder and product theorems, definition and example of Wedderburn polynomials ([LL] §§1-2; [LL2] §§1-3)
   - Koszul algebras: definition, dual of, Hilbert series of ([Froberg] §§1-2)
4. Lie Algebras ([GW] Ch. 1-2) ([Humphreys]§§1-8)
   - Lie group and Lie algebra correspondence
   - classification of finite dimensional semisimple algebras over the complex numbers
   - classification of irreducible representations by highest weight (isomorphism)
   - some explicit examples
   - PBW Theorem

Minor Topic: Hopf Algebras

- Definition of Hopf algebra (coalgebra, bialgebra, antipode)
- Coideals and comodules
- Duality - A^0, C^*
- Definition of integrals and smash product
- H-module algebra and coalgebra
- Fundamental Theorem of Coalgebras

References

Bergman Bergman, George M. "The Diamond Lemma for Ring Theory." Advances in Mathematics, 29 (1978) 178-195.

Froberg Froberg, R. "Koszul Algebras." 1998.

GGRW Gelfand, Israel, Sergei Gelfand, Vladimir Retakh, and Robert Wilson. "Quasideterminants." Advances in Mathematics 193 (2005) 56-141.

GRSW Gelfand, Israel, Vladimir Retakh, Shirlei Serconek, and Robert Wilson. "On a Class of Algebras Associated to Directed Graphs."

GW Goodman, Roe and Nolan Wallach. Representations and Invariants of the Classical Groups. Encyclopedia of Mathematics and its Applications, Cambridge, 1998.

Humphreys Humphreys, James E. Introduction to Lie Algebras and Representation Theory. Springer-Verlag, New York, 1972.

Lam Lam, T.Y. A First Course in Noncommutative Rings. 2nd ed. Springer-Verlag, New York, 2001.

LL Lam, T.Y. and A. Leroy. "Vandermonde and Wronskian Matrices over Division Rings." Journal of Algebra 119, 308-336 (1988).

LL2 Lam, T.Y. and Andre Leroy. "Wedderburn Polynomials over Division Rings, I." Contemporary Mathematics.