

Emilie Hogan's Oral Exam Syllabus

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1 Enumerative Combinatorics

1.1 Generating Functions

Basics: Formal power series, ordinary generating functions, Dirichlet series

Exponential Formula: exponential generating functions, fundamental theorem of exponential generating functions, Lagrange inversion

Rational Generating Functions: Rational power series, polynomials, quasi-polynomials

References: [Wil94], [Zei08], [Sta97]

1.2 Recurrences

Linear Recurrences: c-finite and p-finite, Homogeneous vs. Non-homogeneous, solving techniques

Non-linear Recurrences: Somos sequences, solving techniques

References: [GK82]

1.3 Partially Ordered Sets

Basics: Definition of poset, chain, antichain, graded/ranked posets, Hasse diagrams, union of posets, product of posets

Lattices: Meet and join, lattice definition, complemented lattice, distributive lattices, Birkhoff's representation theorem

Other: Incidence algebra, Möbius inversion, Inclusion-Exclusion

References: [Sta97]

1.4 Impartial Combinatorial Games

Theory: P/N-positions, Nim-Sum, misère play, games on graphs, Sprague-Grundy function, sums of combinatorial games

Examples: subtraction games, Nim, coin turning games, green hackenbush

References:

2 Graph Theory

Basic graph theory: basic graph definitions, trees, bipartite graphs, path and cycles

Matching theory: Hall/König and applications, Tutte's 1-factor theorem, Gallai and Millgram thm

Connectivity: Maders theorem, Mengers theorem

Planarity: Euler's theorem, Kuratowski's theorem, Wagner's theorem

Hamiltonicity: Dirac's theorem, Ore's theorem, Bondy-Chvátal theorem, Hamiltonian cycles and degree sequences (Chvátal's theorem).

Graph Algorithms: Kruskal's, Dijkstra's, Max Flow-Min Cut (Ford-Fulkerson)

Coloring: Vertex coloring (Brook's thm), edge coloring (Vizing's thm), statement of weak/strong perfect graph theorem, edge list coloring

Other: Duality in graph theory

References: [Die05]

3 Hypergeometric Functions

Definitions: Basic definition of hypergeometric series for single variable and multivariable, definition in terms of differential equations for single variable.

Summing: Formulas for the sum of a hypergeometric function when $x = 1$. Explain using Euler integrals, combinatorics, and WZ theory.

Other: q -analogues, difference analogues, A-systems and connections with geometry.

References:

References

[Die05] Reinhard Diestel, *Graph Theory*, Springer-Verlag, Heidelberg, NY, 2005.

[GK82] Daniel H. Greene, Donald E. Knuth, *Mathematics for the Analysis of Algorithms*, Birkhäuser, Boston, MA, 1982.

[Sta97] Richard P. Stanley, *Enumerative Combinatorics Volume I*, Cambridge University Press, 1997.

[Wil94] Herbert S. Wilf, *Generatingfunctionology*, Academic Press, Inc., 1994.

[Zei08] Doron Zeilberger, 'Enumerative and Algebraic Combinatorics,' in *Princeton Companion to Mathematics*, (ed.) Timothy Gowers, Princeton University Press, 2008.