# Syllabus for E. Curry's Oral Qualifying Exam, version 2

**Abstract**

The primary topic for my exam is harmonic analysis and wavelets. The secondary topic is probability theory. References for each topic are given at the end of each section. Supplementary references are given where applicable.

Topic I: Harmonic Analysis, Wavelets, Methods, and Related Material

1. definitions of Fourier coefficients, Fourier series (over $\mathbf{T}$) and transform (over $\mathbf{R}$)
2. summability methods: summability kernels and Fourier multipliers
   (a) partial sums, Dirichlet "kernel" and corresponding multipliers
   (b) Fejer's kernel $\sigma_n$ and corresponding multipliers
   (c) Poisson kernel $P_r$ and corresponding multipliers
   (d) Gauss' kernel $G_\lambda$ (for transforms over $\mathbf{R}$)
   (e) norm convergence of summability methods ($L^1$, general $L^p$)
   (f) pointwise convergence of summability methods
3. uniqueness theorem for Fourier coefficients
4. decay of Fourier coefficients
   (a) Riemann-Lebesgue Lemma
   (b) bound from the $L^1$ norm of a function
   (c) bound from the $L^1$ norm of derivatives if a function is differentiable
   (d) bound from the variance, if a function is of bounded variation
   (e) bound from the modulus of continuity
   (f) bound if a function is in $\mathrm{Lip}_\alpha$
   (g) theorem that if a sequence $\{a_n\}$ is such that $\sum |a_n|^2 = \infty$, then there is no condition on the moduli of the terms that can ensure that $\{a_n\}$ is the sequence of Fourier coefficients on an $L^1(\mathbf{T})$ function

5. effects of rotation, translation, and dilation of Fourier coefficients
6. $L^2$ theory of Fourier series and transforms
   (a) Bessel's Inequality
   (b) Riesz-Fischer Theorem
   (c) Parseval's formula
   (d) Plancherel's formula
7. conditions for existence of Fourier Inversion Formula
8. $L^p$ theory ($1 < p < \infty$) of Fourier series and transforms
   (a) Marcinkiewicz Interpolation Theorem
   (b) convergence of partial sums in $L^p$ norm
   (c) Hilbert transform, principal value
   (d) Hausdorff-Young Theorem
   (e) additional reference: Stein and Weiss. Introduction to Fourier Analysis on Euclidean Spaces. Princeton University Press, Princeton, NJ; 1971.
9. Poisson summation formula
10. Hardy-Littlewood maximal function and related methods
    (a) Vitali Covering Lemma
    (b) maximal function is weak type on $L^1$, bounded on $L^p$ ($1 < p$)
    (c) Calderon-Zygmund decomposition
11. basics of MRA wavelets ($\mathbf{T}$and $\mathbf{R}$)
    (a) wavelets, scaling functions
    (b) multiresolution analyses
    (c) Riesz sequences, Riesz Lemma
    (d) construction of MRA wavelets
    (e) periodic MRA wavelets
    (f) compactly supported MRA wavelets
    (g) smooth MRA wavelets
12. specific wavelets
    (a) Haar wavelets
    (b) Meyer wavelets
    (c) spline wavelets

   (d) Daubechies D3 wavelet
   (e) construction of Daubechies D3 wavelet

13. some function spaces
   (a) $\mathrm{BMO}_p$, BMO
   (b) $H_1^q$, $H_1$
   (c) relation between the two
   (d) $q$-atoms

14. unconditional convergence
   (a) of series
   (b) unconditional bases ($L^1$, general $L^p$)
   (c) Khintchine's inequality

15. Uncertainty Principle
   (a) Heisenberg Inequality (for $f \in L^2$, $a, b$ real, $\int (x-a)^2 |f|^2, \int (\xi - b)^2 |\hat{f}|^2 > \frac{||f||^4_{L^2}}{16\pi^2}$)
   (b) If $f \in L^1$, $|supp(f)||supp(\hat{f})| < \infty$, then $f = 0$.
   (c) $P_A$, $Q_B$ projection operators in time and frequency, resp., $A$, $B$ both of finite measure, then (i) $P_A Q_B$ is a bounded operator from $L^p$ to $L^q$, (ii) $f \neq 0 \in L^2$ $\epsilon-$ concentrated on $A$, $\hat{f}$ $\delta-$ concentrated on $B$, then $1 - \epsilon - \delta \leq$ operator norm of $P_A Q_B$
   (d) primary reference: Folland and Sitaram. The Uncertainty Principle: A Mathematical Survey. J Fourier Analysis and Applications. **3, 3; 1997.**

16. primary references (unless noted otherwise):

   (a) Katznelson. An Introduction to Harmonic Analysis. Dover Publications, Inc., New York; 1976.
   (b) Krantz. A Panorama of Harmonic Analysis. Carus Mathematical Monographs 27. MAA; 1999.
   (c) Wojtaszczyk. A Mathematical Introduction to Wavelets. London Mathematical Society Student Texts 37. Cambridge University Press; 1997.

Topic II: Probability

1. basics
    (a) probability spaces
    (b) random variables
    (c) distribution functions
    (d) expectation
    (e) variance
    (f) moment generating functions
    (g) independence
    (h) convergence (in probability, almost sure, in distribution)
2. properties
    (a) Tchebyshev Inequality
    (b) Chernov Inequality
    (c) Levy Reflection Principle
    (d) Borel-Cantelli Lemma
3. tail stuff
    (a) tail $\sigma -$ algebras
    (b) Kolmogorov's zero-one law
4. convergence theorems
    (a) weak law of large numbers
    (b) strong law of large numbers
    (c) Kolmogorov's Three-Series Theorem
5. conditional expectation, Radon-Nikodym Theorem
6. properties of conditional expectation
    (a) if $X$ is independent of the $\sigma -$ algebra conditioned on
    (b) tower property
    (c) product of random variables, where one is measurable wrt the $\sigma -$ algebra being conditioned on
    (d) basic measure theory properties (positivity, monotone convergence, dominated convergence, Fatou's Lemma, Jensen's Inequality)
7. martingales
8. optional stopping times

9. Optional Stopping Theorem
10. Bernoulli random walks
11. two theorems about stopped Bernoulli random walks
    (a) $\mathbf{P}(\bigcap_{-\infty}^{\infty} T_k < \infty) = 1$
    (b) $\mathbf{P}(T_b < \infty) = (p/q)^b < 1$
12. primary reference: lecture notes from Professor Ocone's class on Probability Theory, Rutgers University, Fall 2000.