

Oral Qual Syllabus

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1 Combinatorics

Basic Enumeration:

counting arguments ([vLW, Chapter 13])

binomial coefficients ([K07, 9/4–9/11])

recurrence relations and generating functions ([vLW, Chapter 14])

inclusion–exclusion ([K07, 9/21–9/25], [vLW, Chapter 10])

Stirling’s formula ([F, p. 52–54], [K07, 9/14])

Set Systems:

Sperner’s Theorem and the LYM Inequality ([Bol, 3], [K07, 10/5–10/9], [vLW, p. 54])

Kruskal-Katona ([Bol, 5], [K07, 11/13]), implies Erdős-Ko-Rado ([Bol, 7])

Erdős-Ko-Rado ([Bol, 7], [K07, 11/9], [vLW, p. 56])

Fisher inequality ([Bol 10.8], [vLW, p. 222])

Frankl-Wilson, Ray-Chaudhuri-Wilson ([FW], [vLW, Thm 19.8, p. 222])

Correlation Inequalities:

Four Functions Theorem ([AS, 6.1], [K07, 12/7], [Bol, 19.1])

FKG Inequality ([AS, 6.2], [K07, 12/4], [Bol, 19.5])

Harris-Kleitman ([AS, 6.3], [K07, 11/30], [Bol, 19.9])

Shepp’s XYZ-theorem ([AS, 6.4], [K07, 12/11], [S])

Ramsey Theory:

Ramsey’s Theorem for graphs and hypergraphs ([K07, 10/19], [vLW, p. 28])

countable and uncountable Ramsey Theory ([Bol, 20], [GRS, 1.1–1.2])

probabilistic lower bounds, upper bounds ([AS, p. 16, 25, 67], [vLW, p. 30])

Van der Waerden’s Theorem ([GRS, 2.1])

Hales-Jewett ([GRS, 2.2])

2 Graph Theory

Matching Theory:

Hall's theorem ([Bol, 2.2], [D, 2.1.2], [K07, 10/12], [vLW, 5])
matching algorithm: Augmenting Paths ([D, 2.1.2, second proof])
König's theorem ([D, Thm 2.1.1], [K07, 10/12])
König is equivalent to Dilworth's Theorem ([K07, 10/5, 10/12])
Gale-Shapley Algorithm for stable matchings ([D, Thm 2.1.4])
Tutte's Theorem on 1-factors ([D, Thm 2.2.1])

Spanning Trees:

Greedy algorithm for maximal spanning trees (matroid result)
Cayley's Theorem, Prüfer Codes ([vLW, 2.1, proof 1])

Planarity:

Euler's Formula ([D, Thm 4.2.9], [vLW, 33.3])
Kuratowski ([D, 4.4], [vLW, 33.2])
Wagner's Theorem ([D, 4.4.6])

Coloring:

Chromatic and Edge Chromatic Numbers
5 color theorem ([D, Prop 5.1.2], [vLW, 33.6])
Brook's Theorem ([D, Thm 5.2.4], [vLW, 3.1])
Vizing's Theorem ([D, Thm 5.3.2])

Extremal Problems:

Turan's Theorem ([vLW, 4], [D, p. 165], [AS, p. 91-92])
Statement of Regularity Lemma ([D, p. 176])
Erdős-Stone Theorem ([D, 7.5])
Triangle Removal Lemma (given Regularity Lemma)
Proof of Roth's Theorem (given Regularity Lemma)

Matroids:

Definitions, multiple characterizations ([Wal, 1])
Greedy Algorithm ([Wal, 19.1])
Covering and Packing Theorems ([Wal, 8.1-8.4])
Matroid Intersection Theorem, implies König's Theorem ([Wal, 8.5])
Transversals and generalizations of Hall's Theorem ([Wal, 7.1-7.4])

3 Probabilistic Methods

Basics:

Linearity of Expectation ([AS, Chapter 2], [K07,10/23])

Bonferroni Inequalities ([Dur, I.1.3], [K07, 9/25])

common distributions ([Dur, I.1.5-1.7], [K07,9/25])

conditional probability, law of total probability ([K07, 11/9])

Chernoff bounds ([AS, Apdx. A], [K07, 11/2])

Chebyshev Inequality ([AS, p. 41], [Dur, I.3.7], [K07, 10/26])

Alteration Method:

general procedure ([AS, 3.1])

basic examples:

independent sets, packing, triangles in the unit square ([AS, 3.2-4])

application to Property B (hypergraph coloring) ([AS, 3.5])

Second Moment Method

general procedure ([AS, 4.1], [K07, 10/26])

application to threshold functions ([AS 4.4], [K07, 10/26-10/30])

Moment Method for eigenvalues of random matrices ([V08, 2/18])

Lovasz Local Lemma:

Symmetric and general versions ([AS, 5.1], [K07, 10/30-11/2])

Ramsey lower bounds ([AS, 5.3])

Hypergraph Coloring ([AS, 5.2], [K07, 10/30])

Martingales:

Definitions, Azuma's Inequality ([AS 7.1-2], [V08, 4/3, 4/10])

application to chromatic number ([AS, 7.3])

Talagrand's inequality ([AS, 7.5-7.6], [V08, 3/10, 3/27, 4/3])

comparing Talagrand's and Azuma's inequalities ([AS, 7.7], [V08, 3/10])

longest increasing subsequence problem ([AS, 7.7])

4 Probability Theory

Basics, Large Number Laws:

probability spaces ([Dur, I.1])

random variables([Dur, I.2])

expectation ([Dur, I.3])

independence of random variables ([Dur, I.4])

weak law of large numbers ([Dur, I.5])
 Borel-Cantelli lemma ([Dur, I.6])
 convergence concepts for random variables ([Dur, I.7])
 Kolmogorov's 0-1 law ([Dur, I.7.1])
 Kolmogorov's three series theorem ([Dur, I.7.4])
 strong law of large numbers ([Dur, I.8])
Central Limit Theorem:
 De Moivre-Laplace theorem ([Dur, II.1])
 weak convergence and convergence in distribution ([Dur, II.2])
 characteristic functions ([Dur, II.3])
 continuity theorem, characteristic functions ([Dur, II.3.6])
 Lindeberg-Feller central limit theorem ([Dur, II.4])
 Berry-Esseen Theorem ([Dur, II.4])
Martingales:
 conditional expectation ([Dur, IV.1])
 definition of (sub)(super) martingales ([Dur, IV.2])
 stopping times ([Dur, III.1])
 application to random walks ([Dur, III])
 Doob's upcrossing inequality ([Dur, IV.2.9])
 martingale convergence theorems ([Dur, IV.2.10])

References

- [AS] Alon, Noga; Spencer, Joel H. *The probabilistic method*. Second edition. Wiley-Interscience Series in Discrete Mathematics and Optimization. Wiley-Interscience [John Wiley & Sons], New York, 2000. xviii+301 pp.
- [Bol] Bollobás, Béla. *Combinatorics: Set systems, hypergraphs, families of vectors and combinatorial probability*. Cambridge University Press, Cambridge, 1986. xii+177 pp.
- [D] Diestel, Reinhard. *Graph theory*. Third edition. Graduate Texts in Mathematics, 173. Springer-Verlag, Berlin, 2005. xvi+411 pp.