

1 Probability Theory

Probability Spaces

- Algebras and σ -algebras of collections of sets (Review)

- Probability spaces

- Probability spaces; elementary examples, coin tossing

- The monotone class theorem

- Independence and product spaces

Random Variables, Distribution Functions, and Expectation

- Random variables and distribution functions

- Laws of random variables and distribution functions

- Expected values

- Independence of random variables

- Some basic probability laws

- Convergence concepts for random variables

Large Number Laws for Sequences of Random Variables

- The weak law of large numbers

- The Borel-Cantelli lemma

- Strong large number laws

- Convergence of infinite series of independent random variables and applications

Convergence in Distribution and the Central Limit Theorem

- Motivation: the central limit problem

- Weak convergence and convergence in distribution

- Characteristic functions; uniqueness and smoothing

- The continuity theorem

Conditional Expectation

- Definition of conditional expectation

- Properties of conditional expectation

Martingale Theory in Discrete Time

- Definitions and examples

- Stochastic integrals in discrete time and a fundamental theorem of Martingale theory

- Optional stopping of Martingale

- Martingale inequalities

- Martingale convergence theorems

2 Combinatorics I & II

Basic Enumeration

- Basic Counting
- Recurrence Relations
- Generating Functions
- Principle of Inclusion–Exclusion

Partially Ordered Sets, Lattices

- Dilworth’s Thm
- Sperner’s Thm
- Geometric and Distributive Lattices
- Möbius Functions

Hypergraphs

- Block Designs
- Fisher’s Ineq. & Related
- Erdős–Ko–Rado Thm
- Kruskal–Katona
- Hamming Cube & Isoperimetry
- Turán’s Thm & Erdős–Stone
- Baranyai

Ramsey Theory

- Infinite Ramsey
- Finite Ramsey
- Van der Waerden’s Thm, Szemerédi’s Thm

3 Graph Theory

Matching Theory

- Hall/König & Applications
- Max Flow Min Cut
- Tutte’s 1–factor Thm

Connectivity

- 2–connected graphs
- Menger’s Thm

Graph Coloring

- Vertex Colorings
- Edge Colorings

Planar Graphs

Matroids

4 Probabilistic Methods

Lin. of Expectation & “Alterations”

Second Moment Method

Lovász Local Lemma

Poisson Paradigm & Janson Ineq.

Correlation Inequalities