Syllabus for Oral Exam

WANG, Yu

Major topic: Partial Differential Equation

1. Four Important Linear PDEs

    1.1Transport Equation

    1.2Laplace's Equation

    Fundamental solution

    Mean-value formula

    Properties of harmonic functions (Maximum principles, smoothness)

    Green's function

    1.3Heat Equation

    Fundamental solution

    Mean-value formula

    Maximal principle (for bounded domain; for Cauchy problem)

    Energy methods (for uniqueness for initial/boundary-value problem; for backward uniqueness)

    1.4Wave Equation

    d'Alembert's formula

    Solution by spherical means

    Energy methods (for uniqueness for wave equation)

    1.5Schrodinger Equation

    Derivation of its fundamental solution

2. Sobolev Spaces

2.1Holder spaces

2.2Sobolev spaces

Weak derivatives, definition of Sobolev spaces, elementary properties

2.3approximation

2.4extentions

2.5Sobolev inequalities

Gagliardo-Nirenberg-Sobolev inequality

Morrey's inequality

General Sobolev inequality

2.6Compactness

3. More General Second-Order Elliptic Equations

3.1Existence of weak solutions

Definition of weak solution

Lax-Milgram Thm

Energy estimates

Fredholm alternatives

3.2Regularity

Interior regularity, boundary regularity

3.3Maximum principles

Weak maximum principle

Strong maximum principle

Minor topic: Numerical Solution for PDEs

1. Approximation of function by polynomial and piecewise polynomial

Newton interpolation

Lagrange interpolation

Application to numerical differentiation and numerical quadrature

Error Estimates

2. Finite Difference Method for elliptic PDEs and heat PDEs

   discrete maximum principle

   discrete Green's function

   error estimates

3. Finite Element Methods for PDEs

   triangulation of domain

   barycentric coordinates

   variational formulations of elliptic PDE (standard Galerkin; mixed finite element methods; nonconforming methods)

   error estimates in H^1 and L^2

   solution of the discrete equations