Topics for oral qualifying exam for Fei Qi
Fall, 2014

Major topic: Vertex operator algebras

1. Definitions and properties.
   (a) Formal calculus.
   (b) The notions of vertex algebra and of vertex operator algebra, and basic properties.
   (c) Rationality, commutativity and associativity; equivalence of various formulations, including "weak" formulations.
2. Representations of vertex (operator) algebras.
   (a) The notion of module and basic properties.
   (b) Weak vertex operators.
   (c) The structure of the canonical weak vertex algebra. Local subalgebras and vertex subalgebras of the canonical weak vertex algebra.
   (d) The equivalence between modules and representations.
   (e) General construction theorems for vertex (operator) algebras and modules.
3. Examples of vertex (operator) algebras and modules.
   (a) Vertex (operator) algebras and modules based on the Virasoro algebra.
   (b) Vertex (operator) algebras and modules based on affine Lie algebras.
   (c) Vertex (operator) algebras and modules based on Heisenberg Lie algebras.
   (d) Vertex (operator) algebras and modules on even lattices.
   (e) Vertex operator construction of the affine Lie algebras corresponding to A_n, D_n and E_n.

4. Affine Lie algebras

   (a) Classification of affine Lie algebras, twisted and untwisted.

   (b) Explicit realization of affine Lie algebras.

   (c) Affine root systems and Weyl groups.

5. The geometry of vertex operator algebras

   (a) The moduli spaces of spheres with tubes, the sewing operation, examples for the sewing operation.

   (b) The geometric interpretations of vertex operators and vacua, the geometric meanings of commutativity, associativity, skew-symmetry, L(1)-derivative property, vacuum property and creation property.

   (c) The notion of geometric vertex operator algebra and the statement of the isomorphism theorem.

   (d) Determinant lines of Fredholm operators, determinant lines over Riemann surfaces with parametrized boundaries, relation to the central charges of vertex operator algebras.

Minor topic: Quasideterminants and noncommutative symmetric functions

1. General theory of quasideterminants.

2. The Heredity principle and its applications.

3. Noncommutative symmetric functions.

4. The Viète theorem and factorizations of noncommutative polynomials.

References

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[Hua] Y.-Z. Huang, *Two-dimensional conformal geometry and vertex operator algebras*, Progress in Mathematics, Vol. 148, Birkhäuser, Boston, 1997.

[Hum] J. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Second Printing, Revised, Springer-Verlag, 1972.

[KL] D. Krob and B. Leclerc, Minor identities for quasi-determinants and quantum determinants, *Comm. Math. Phys.* Volume 169, Number 1 (1995), 1-23.

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