Oral Exam

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Part 1: Analysis

1. Sobolev spaces
   a) Definition of weak derivative and Sobolev spaces
   b) Approximation by smooth functions
   c) Extensions
   d) Traces
   e) Sobolev inequalities
      - Gagliardo-Nirenberg-Sobolev inequality
      - Morrey's inequality
      - General Sobolev inequalities
   f) Compactness
   g) Poincare's inequalities
   h) Difference quotients
   i) The space H^(-1)

2. Second-Order Elliptic Equations
   a) Definitions
      - Elliptic equations
      - Weak solutions
   b) Existence of weak solutions
      - Lax-Milgram Theorem
      - Energy estimates
      - Fredholm alternative
   c) Regularity

3. Optimal Mass Transportation
   a) Kantorovich Duality
      - Fenchel-Rockafellar duality
      - Kantorovich Duality
   b) Optimal transportation theorem for quadratic cost
      - Knott-Smith optimality criterion
      - Brenier's theorem
   c) Sobolev inequality

Part 2: Riemannian Geometry

1. Connections
   a) Affine connections
   b) Riemannian connections
   c) Covariant derivative along a curve

2. Geodesics
   a) Geodesic equation
   b) Geodesic flow
   c) Exponential map
   d) Gauss lemma
   e) Minimizing properties

3. Curvature
   a) Curvature
   b) Bianchi identities
   c) Sectional curvature
   d) Ricci curvature
   e) Scalar curvature

4. Jacobi fields
   a) Jacobi equation
   b) Conjugate points

5. Completeness
   a) Hopf-Rinow theorem
   b) Hadamard theorem

References

Manfredo P. Do Carmo, *Riemannian Geometry*, Birkäuser, 1992.

Lawrence C. Evans, *Partial Differential Equations*, AMS, 1998.

Cédric Villani, *Topics in Optimal Transportation*, AMS, 2003.