

Oral Exam Syllabus

Student: German Enciso
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Prof. Eduardo Sontag
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Prof. Richard Wheeden
Date: Monday, December 16th
Time & Place: 10 am, Hill 425

Subject I: Harmonic Analysis

- 1) The classic construction of the Fourier Transform.
- 2) The Lebesgue set, and the maximal function.
- 3) Marcinkievitch interpolation theorem.
- 4) Calderón-Zygmund decomposition.
- 5) Singular integral theorems.
- 6) Hilbert and Riesz Transforms.
- 7) Poisson Kernel.
- 8) Spherical Harmonics

Sources: Stein, Elias, *Singular Integrals and Differentiability Properties*, Princeton University Press 1970.

Subject II: Numerical solutions to PDEs, with background in theoretical PDEs

Background:

- 1) Maximum principle.
- 2) Harnack Inequality.
- 3) Mean Value Property.
- 4) Green's Function on the unit ball
- 5) Poisson Formula

Numerical Methods:

- 6) Discrete maximum principle.
- 7) The discretisation of the problem

$$\begin{cases} -\Delta u = f & \text{in } \Omega \\ u = \phi & \text{in } \partial\Omega \end{cases}$$

and its solution using finite difference methods.

8) Variational principles.

9) Finite elements solution of the above problem.

Sources: Evans, L.C., *Partial Differential Equations*, American Mathematical Society, 1998; Braess, Dietrich, *Finite Elements*, Cambridge University Press 2001.