Oral Qualifying Examination Syllabus
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1. Commutative Algebra

1.1. Basic Constructions.

- Localization
- Hom, Tensor, Ext, and Tor
- Associated Primes and Primary Decomposition of Modules
- Integral dependance and Integral Extensions
- Graded Rings and Modules
- Filtrations
  - I-stable filtration
  - Associated Graded Ring and Module
  - Blow-up Algebra, Rees Algebra
  - Artin-Rees Lemma
  - Krull Intersection Theorem
- Completions and Hensel's lemma

1.2. Dimension Theory.

- Dimension Zero
- Principal Ideal Theorem and System of Parameters
- Noether Normalization
- Finiteness of Integral Closure

1.3. Cohen Macaulay Rings.

- Regular Sequences
- Grade, Depth, and Projective Dimension
  - Auslander-Buchsbaum Formula
- Cohen-Macaulay Rings and Modules
- Regular Local Rings
- Normal Rings
  - R1 and S2 conditions
- Complete Intersections

2. Computational Geometry

2.1. **Monomial Ideals.**
- Dickson's Lemma
- Primary Decompostition
- Cellular Resolution
- Multi-gradings
- Borel-Fixed Ideals

2.2. **Gröbner Bases.**
- Monomial Orders
- Weight Orders
- Buchberger's Criterion
- Buchberger's Algorithm
- Free Resolutions
    - Schreyer's Method for Computing Syzygies
    - Hilbert Syzygy Theorem
    - Minimal Free Resolutions
- Universal Gröbner Basis
- State Polytope and Grobner Fan

2.3. **Hilbert Functions.**
- Bezout's Theorem
- Macaulay's Theorem
- Gotzmann's Regularity and Persistence Theorems

2.4. **Varieties.**
- Nullstellensatz
- Ideal-Variety Dictionary
- Elimination and Projection
- Toric Ideals and Varieties
    - Toric Ideal is Generated by Binomials
    - Compute Hilbert Basis for Affine Toric Variety

2.5. **Polytopes.**
- Equivalent Characterizations
- Simple and Simplicial Polytopes
- Cyclic Polytope and Gale's Evenness Condition
- Simplicial Complex
- Shellability
- Upper Bound Theorem for Convex Polytopes