Oral Qual Syllabus

1. Combinatorics

   - Basic Enumeration: Counting arguments, recurrence relations, generating functions, binomial coefficients, principle of inclusion-exclusion, Stirling's formula

   - Set Systems: Sperner's Theorem, Erdős-Ko-Rado, Kruskal-Katona, Fisher's and generalized Fisher's Inequalities, uniform and nonuniform Raychaudhuri-Wilson, Frankl-Wilson, LYM Inequality, Harper's Theorem, Baranyai's Theorem

   - Lattices and Posets: distributive and geometric lattices, linear extensions of posets, Dilworth's Theorem, Birkhoff Representation Theorem, Möbius Inversion, Weisner, Dowling-Wilson

   - Correlation Inequalities: Ahlswede-Daykin, FKG, Harris-Kleitman, XYZ Theorem

   - Algebraic Methods: Inclusion matrices and application to Gasparian's proof of the weak perfect graph theorem, Combinatorial Nullstellensatz, Chevalley-Warning and application to Alon-Dubiner, Erdős-Ginsberg-Ziv

   - Ramsey: Ramsey's Theorem, Infinite Ramsey, König's Infinity Lemma, probabilistic lower bounds, Hales-Jewett, van der Waerden, statement of Szemerédi's Theorem

   - Discrepancy: Beck-Fiala

2. Graph Theory

   - Matchings: König's Theorem, Hall's Theorem, Tutte's 1-factor Theorem, stable matchings, polytopes

   - Connectivity: Kruskal's Algorithm for minimum weight spanning tree, structure of 2-connected graphs, Menger's Theorem, Max-Flow-Min-Cut Theorem

   - Planarity: Euler's Formula, Kuratowski's Theorem, Wagner's Theorem

- Colorings: 5-Color Theorem, Brooks' Theorem, Vizing's Theorem, Thomasson's 5-list-coloring of planar graphs, Hadwiger's conjecture, perfect graphs, Lovász' proof of the Weak Perfect Graph Theorem, statement of the Strong Perfect Graph Theorem

- Extremal: Turán's Theorem, statement of the regularity lemma, Erdős-Stone, Chvátal-Rödl-Szemerédi-Trotter

3. Probabilistic Methods

- Basics: Linearity of Expectation, Bonferroni inequalities, Chebyshev's Inequality, Chernoff Bounds, alterations

- Second Moment Method: general procedure, threshold function for containing a given subgraph

- Lovász Local Lemma: general and symmetric versions, application to hypergraph discrepancy, application to Ramsey lower bounds, application to Latin transversals

- Poisson Paradigm: Janson inequalities and application to the number of triangles in $G_{n,p}$, Brun's sieve and application to the number of isolated vertices in $G_{n,p}$

- Martingales: vertex and edge exposures, Azuma's Inequality and application to $\chi(G)$

- Random Graphs: $G_{n,p}$ v. $G_{n,M}$, monotone properties, existence of threshold functions, Bollobás-Thomason, probabilistic refutation of the Hájos conjecture

- Entropy: basic properties, Shearer's Lemma, application to Minc Conjecture

4. Probability Theory

- Probability Spaces and Random Variables: algebras and σ-algebras, probability spaces, monotone class theorem, independence and product spaces, random variables, distribution functions, expectation, independence of random variables, convergence concepts for random variables, Kolmogorov's Zero-One Law

- Large Number Laws: weak law of large numbers, Borel-Cantelli Lemma, strong law of large numbers, Kolmogorov's three series theorem

- Stationary Processes: stationarity, measure-preserving transformations, Birkhoff 's ergodic theorem, ergodic theorem for stationary processes

- Central Limit Theorem: De Moivre-Laplace, weak convergence and convergence in distribution, characteristic functions, continuity theorem, Lindeberg-Feller

- Martingales: conditional expectation, definition of (sub)(super) martingales, stopping times, optional stopping theorems, application to random walks, Doob's martingale inequalities, Doob's upcrossing inequality, uniform integrability, martingale convergence theorems, Levy's upward convergence theorem