

Syllabus for Oral Qualifying Exam

Hanlong Fang

Committee: Sagun Chanillo, Xiaojun Huang, Feng Luo, Jian Song.

I. Several Complex Variables

1. Cauchy integral formula and its applications:

- Cauchy integral formula in polydiscs, Cauchy estimates;
- Hartogs extension Theorem;
- Bochner's extension Theorem.

2. Subharmonicity and convexity:

- Properties of subharmonic functions and plurisubharmonic functions;
- Domain of holomorphy, Continuity Principle;
- Reinhardt domain and its holomorphic convex hull;
- Tube domain and its holomorphic convex hull;
- Pseudoconvexity, Levi pseudoconvexity;
- Oka's Theorem.

3. Coherent Sheaves and Complex Analytic Spaces

- Oka's Coherence Theorem for Structure Sheaves;
- Oka-Cartan Coherence Theorem of the Ideal Sheaf;
- Coherence of the Normalization Sheaf ;
- Coherence of Direct Image Sheaves in proper mapping ;
- Cartan A, Cartan B and Stein Spaces;
- Cartan-Serre VS Andreotti-Grauert (compared with Grothendieck's Vanishing Theorem)

3. L^2 theory for $\bar{\partial}$ on pseudoconvex domains:

- L^2 Existence Theorem for $\bar{\partial}$ operator;
- The Levi problem.

4. Compact Algebraic Surface:

- Intersection Numbers;
- Riemann-Roch for curves and surfaces;
- Blow up a point
- Ample divisor, Nef divisor and Big divisor;
- Resolution of singularities of embedding curves
- Riemann-Hurwitz formula for surfaces and curves
- Castelnuovo (-1)-curves contracting theorem

II. Second Order Elliptic Partial Differential Equations

1. Sobolev Embedding Theorem and Poincaré Inequality
2. Schauder $C^{2,\alpha}$ estimate
3. Calderon-Zygmund Kernels and $W^{2,p}$ estimates

III. Geometry and Topology

1. The Fundamental Group and Covering Spaces
2. Hyperbolic metric on the upper half plane and on the disc
3. Uniformization Theorem for Riemann surfaces
4. Gauss-Bonnet for polygons in upper half planes

References

- [HL] Qing Han and Fanghua Lin, Second Elliptic PDEs
- [GR] Hans Grauert and Reinhold Remmert, Coherent Analytic Sheaves
- [H1] Xiaojun Huang, Lecture notes for Several Complex Variables (unpublished)
- [H2] Xiaojun Huang, Lecture notes for One Complex Variables (unpublished)

- [L] Feng Luo, Tsinghua Summer School Lecture notes for the course Hyperbolic Geometry(unpublished)
- [T] Shengli Tan, Algebraic Surfaces (unpublished Chinese notes)