

Oral Qual Syllabus

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1 Algebraic Geometry

1. **Sheaves:** Definition, presheaves and associated sheaves, morphisms, stalks, pull-back and push-forward

2. **Schemes:** Affine schemes, Proj, structure sheaf, scheme associated to a variety

3. **First properties of schemes:** Reduced, integral, Noetherian schemes, morphisms of finite type, finite morphisms, closed immersions, dimension, fibre products

4. **Separated and proper morphisms:** Definition of separated and proper morphisms, valuative criteria, projective morphisms, reduced structure of closed subsets, scheme-theoretic image, constructible sets

5. **Coherent and quasi-coherent sheaves:** Definition of \mathcal{O}_X -modules, quasi-coherent and coherent sheaves, constructions on \mathcal{O}_X -modules, invertible sheaves, vector bundles

6. **Divisors:** Weil divisors, Cartier divisors, equivalence for locally factorial schemes, invertible sheaves

7. **Projective morphisms:** Criteria and characterization of projective morphisms, ample and very ample line bundles, blowups

8. **Differentials:** Derivations, module of relative differential forms, sheaves of differentials and nonsingularity, tangent sheaf, canonical sheaf, geometric genus

2 Cohomological methods in algebraic geometry

1. **Derived functors:** Abelian categories, complexes, derived functors, δ -functors

2. **Cohomology of sheaves:** The category of sheaves of \mathcal{O}_X -modules has enough injectives, some basic vanishing theorems

3. **Cohomology of a Noetherian Affine Scheme:** vanishing of higher cohomology of quasi-coherent sheaves and Serre's criterion for being affine in terms of vanishing of cohomology

4. **Čech cohomology:** definition, isomorphism with regular cohomology for a noetherian separated scheme

5. **Cohomology of projective space:** calculation using Čech cohomology, Serre vanishing, cohomological criterion of ampleness
6. **Ext Groups and sheaves:** definition and basic properties
7. **Serre duality:** dualizing sheaf and duality for a projective scheme, invertibility of dualizing sheaf for l.c.i., isomorphism between dualizing sheaf and canonical sheaf for nonsingular projective variety
8. **Higher direct images:** definition and coherence for projective morphisms
9. **Flat morphisms:** definition, properties, cohomology commutes with flat base change, flat families
10. **Smooth morphisms:** definition, properties, generic smoothness, Kleiman's theorem on transversality, étale morphisms
11. **The Theorem on Formal Functions:** Zariski's Main Theorem and Stein Factorization
12. **Spectral Sequences:** Leray spectral sequences

3 Intersection Theory

1. **Rational equivalence:** cycles, definitions of rational equivalence, proper push-forward, flat pull-back, open-closed exact sequence, affine bundles
2. **Divisors:** Cartier and Weil divisors, line bundles and pseudo-divisors, intersection with divisors, chern class of line bundles
3. **Vector Bundles and Chern classes:** segre classes, chern classes, rational equivalence on bundles
4. **Cones and Segre classes:** segre class of a cone, segre class of a subscheme, multiplicity along a subvariety
5. **Deformation to the Normal Cone:** deformation and specialization to the normal cone
6. **Intersection Products:** basic construction, refined Gysin homomorphism, excess intersection formula, commutativity, functoriality, l.c.i. morphisms, blow-up formula
7. **Intersections on non-singular varieties:** refined intersections, intersection multiplicities, intersection ring
8. **RR for Non-singular varieties:** GRR and HRR

4 Complex Algebraic Surfaces

1. **Cohomology:** RR, Noether's formula, and the genus formula
2. **Del Pezzo surfaces:** definitions and blow-ups
3. **K3 surfaces:** definition, cohomology, periods, polarization, examples
4. **Enriques surfaces:** examples and correspondence with K3 surfaces
5. **ADE singularities:** quotients, equations, and resolutions

5 References

1. Hartshorne, *Algebraic Geometry*
2. Griffiths and Harris, *Principles of Algebraic Geometry*
3. Fulton, *Intersection Theory*
4. Beauville, *Complex Algebraic Surfaces*
5. Dolgachev and Cossec, *Enriques Surfaces I*