Committee: Professors Kahn-Saks-Szemerédi-Vu (chair).

1. Arithmetic Combinatorics

1.1. Some definitions and results.

(1) Sum set estimates: doubling constant, additive energy, Ruzsa-distance, covering lemma.
(2) Additive geometry: Blichfeldt's theorem, Minkowski' first theorem, Minkowski's second theorem.
(3) Discrete Fourier analytic:
  (a) Bilinear form, Fourier transformation, Plancherel's identity, inversion, convolution.
  (b) Linear Bias: small uniformity norm implies large sumsets; sum of squares, Bourgain's "$A.A + A.A + A.A$"-theorem, Polya-Vinogradov-type inequality .
  (c) Structure of Bohr neighborhoods in $\mathbf{Z}_N$: size bound (Kronecker' s approximation), relation to proper arithmetic progression.
  (d) Spectrum of an additive set.

1.2. Structure of sets with small doubling constant.

(1) Doubling constant of a set of high rank (Freiman's lemma).
(2) Freiman's isomorphism: passing from torsion-free group to $\mathbf{Z}$, and from $\mathbf{Z}$ to $\mathbf{Z}/p\mathbf{Z}$.
(3) Ruzsa's proof of the Freiman's inverse theorem (for $2A - 2A$); reduction the rank to $\sigma[A] - 1$.

1.3. Sums and products of a set.

(1) Lower bound for the crossing number, Szemerédi-Trotter's theorem. Sum-product in $\mathbf{R}$.
(2) Quotient set, Katz-Tao's theorem, Freiman's theorem for sum-product, sum-product estimate.

1.4. Some graph theoretic results.

(1) Path of length two and three in dense graphs; Gowers'proof of the Balog-Szemerédi's theorem.
(2) Magnification ratio, commutative property and Pünnecke's graph of order k. Plünnecke-Ruzsa's theorem and deduction for sumset estimates of type $kA \pm lA$.

1.5. **Szemerédi's theorem for $k = 3$.** (Roth's theorem)

(1) Behrend's lower bound for $r_3(N)$.
(2) The small torsion case: $r_3(\mathbf{Z}_p^n) = O(p^n/n)$.
(3) The integer case: $r_3(N) = O(\frac{N}{(\log\log N)^c})$.
(4) Szemerédi's regularity lemma for graph, removal lemma; existence of isosceles triangle in the lattice, deduction for arithmetic progression of length 3.

1.6. **Some advanced topics.** (very rough sketch)

(1) Gowers' norm (U_k), relation to arithmetic progressions in a set; U_2 is equivalent with the uniformity norm.
(2) Quadratic analysis approach for Szemerédi's theorem ($k = 4$).
(3) Furstenberg's infinitary ergodic approach for general k: measure preserving system, multiple recurrence, correspondence principle.

Ref: Terence Tao - Van H. Vu: Additive combinatorics.

2. Discrete Mathematics

2.1. **Combinatorics and Graph theory.**

(1) Basic Enumeration: Basic counting, representations of permutations, recurrence relations, generating functions, principle of inclusion-exclusion.
(2) Set system: Erdős-Ko-Rado's theorem, Sperner's theorem and (real) Littlewood-Offord-Erdős's theorem; LYM inequality, Kruskal-Katona's theorem, Bárányai's theorem.
(3) Linear algebraic method: Uniform, non-uniform Ray-chaudhuri-Wilson's theorem; modular Frankl-Wilson's theorem; Fisher's and Generalized Fisher's inequality,
(4) Other algebraic method: combinatorial nullstellensatz and application to Cauchy-Davenport's theorem. Chevalley-Warning's theorem and application to Erdős-Ginzburg-Ziv's theorem.
(5) Infinite Ramsey, finite Ramsey: Ramsey's theorem for coloring, Schur's theorem, Hales-Jewett theorem, Vander Waerden's theorem.
(6) Dilworth's theorem, König' s Lemma, Hall's Theorem, Tutte's 1-factor Theorem, Menger's Theorem.
(7) Euler's formula, Kuratowski' theorem, 5-color theorem, Brook's Theorem, Galvin's theorem, Lovász' theorem on perfect graphs.
(8) Turán's theorem, Erdős-Stone's theorem, statement of the Hajnal-Szemerédi's theorem.

2.2. **Probabilistic method.**

(1) Linear of expectation and the second moment method; diagonal Ramsey, sum-free subset, threshold for balanced graphs.
(2) Deletion method: large girth large chromatic number.
(3) Lovász local lemma, triangle-free graph with small independent set.
(4) Chernoff's inequality, Azuma's inequality; vertex exposure and concentration of chromatic number.
(5) Poisson Paradigm, Janson inequality.
(6) Clique and chromatic number of $G(n, \frac{1}{2})$.

Ref.: Alon-Spencer: The probabilistic method, Bollobás: Combinatorics.