Oral Exam Syllabus

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Part I: Partial Differential Equations

(1) Laplacian Equation
- Fundamental solution.
- Properties for harmonic functions: Mean-value formulas, Maximum principle, Regularities, Liouville's theorem, Harnack's inequality.
- Green's function.
- Variational method for the Dirichlet's principle.

(2) Heat Equation
- Fundamental solution.
- Mean-value formulas, Maximum principle, Regularities.
- Backward Uniqueness.

(3) Wave Equation
- Solution for homogenous and nonhomogenous equations (d'Alembert's formular, Kirchhoff's and Poisson's formulas, etc.)
- Energy methods.

(4) Sobolev Space
- Basic properties.
- Approximations, Extensions, and Traces.
- Gagliardo-Sobolev-Nirenberg inequality, Morrey's inequality, Poincare's inequality.
- Compact imbedding.

(5) Second-Order Elliptic Equations
- Variational formulations and existence of weak solutions.
- Regularities.
- Maximum principle, Harnack's inequality.
- Eigenvalues and eigenfunctions.

Part II: Functional Analysis

(1) Banach Space
- Hahn-Banach Theorem, Separation of convex sets.
- Conjugate convex functions.
- Baire Category theorem, Uniform bounded principle, the open-mapping theorem and closed graph theorem.
- Weak and Weak* topology, Reflexivity, Separability.

(2) Hilbert Space
- Projection onto a convex set.
- Riesz representation Theorem.

- The theorems of Stampacchia and Lax-Milgram.
- Hilbert sums and orthonormal bases.

(3) Compact Operator
- Fredholm alternative.
- The spectrum of compact operators.
- Spectral decomposition of self-adjoint compact operators.

(4) Sobolev functions in real line.
- The properties of $W^{1,p}(I)$.
- Variational formulation and spectrum analysis for certain ODE problems.

References

[1] Haim Brezis, *Functional Analysis, Sobolev Spaces, and PDEs* .

[2] Lawrence C. Evans, *Partial Differential Equations.*