Oral Exam Syllabus

HUMBERTO MONTALVÁN-GÁMEZ

Committee: Profs. József Beck (chair), János Komlós, Stephen Miller, Doron Zeilberger

1 Combinatorics, Graph Theory, & Probabilistic Methods

1.1 Combinatorics

Basics. Pigeonhole principle; counting arguments; generating functions; recurrence relations; inclusion-exclusion formula; binomial coefficients; Stirling numbers; Bell numbers; Catalan numbers; Fibonacci numbers.

Partitions and Tableaux. Ferrer diagrams; Euler's pentagonal number theorem; standard Young tableaux; the hook length formula.

Set systems. Sperner's theorem; LYM inequality; Littlewood-Offord problem; Kruskal-Katona theorem; Erdös-Ko-Rado theorem.

Ramsey theory. Ramsey's theorem for graphs and hypergraphs; countable and uncountable Ramsey theory; König's lemma; van der Waerden's theorem.

Hypergeometric functions and identities. Sister Celine's algorithm; the fundamental theorem of hypergeometric series; Gosper's algorithm; Zeilberger's Algorithm; the WZ method.

1.2 Graph Theory

Hamiltonian cycles. Dirac's theorem; Ore's theorem.

Matching. Hall's theorem; König's theorem; Berge's theorem; Tutte's theorem.

Connectivity and spanning trees. Menger's theorem; max flow/min cut theorem; Prim's algorithm; Kruskal's algorithm; Dijkstra's algorithm.

Planarity. Euler's formula; Kuratowski's theorem; Wagner's theorem.

Coloring. Chromatic and edge chromatic numbers; Brook's theorem; Vizing's Theorem; 5-color theorem, perfect graphs.

Extremal problems. Turán's theorem; statement of regularity lemma; Erdös-Stone theorem.

Random graphs. Notion of a random graph; properties of almost all graphs.

1.3 Probabilistic Methods

Basics. Probability spaces and random variables; binomial, Poisson, and normal distributions; linearity of expectation; Bonferroni inequalities; Chernoff bound.

Alteration method. General idea; application to existence of graphs with high chromatic number and high girth.

Second moment method. General procedure; Chevyshev's inequality; application to threshold function for having a graph as a subgraph.

Lovász local lemma. Symmetric and general versions; application to Latin traversals.

Poisson paradigm. Janson inequalities; application to number of triangles in $G_{n,p}$.

Martingales and tight concentration. Azuma's inequality; vertex and edge exposure; applications to concentration of chromatic number; Talagrand's inequality; comparing Azuma's and Talagrand's inequalities.

2 Diophantine Approximations

Diophantine equations. Pythagorean triples; method of infinite descent; Pell's equation.

Approximability by rationals. Farey sequence; Dirichlet's theorem; Kronecker's theorem; Hurwitz's theorem; order of approximation; Khintchine's theorem;

Continued fractions & improved approximation theorems. Continued fraction expansion of e; law of best approximations; Vahlen's theorem; Borel's theorem; Hurwitz's theorem; Lagrange spectrum & Markov's chain.

Irrational & trascendental numbers. Irrationality of e, $e^{m/n}$, π, and $\zeta(3)$; transcendence of e and π; Liouville's theorem; Thue equation; Sigel's lemma; Roth's theorem.

Geometry of numbers. Minkowski's theorem on convex sets; sums of squares; lattice packings.

3 References

3.1 References for 1.1 Combinatorics

Bollobás, Béla. *Combinatorics: Set systems, Hypergraphs, Families of Vectors and Combinatorial Probability.*

Kahn, Jeffrey. *Notes from the course* Combinatorics I, *Rutgers University, Fall 2007 (taken by Humberto Montalván-Gámez).*

Petkovšek, Marko; Wilf, Herbert S.; Zeilberger, Doron. *A=B.*

Stanley, Richard P. *Ennumerative Combinatorics.* Second Edition.

van Lint, J. H.; Wilson, R. M. *A course in combinatorics.* Second edition.

Zeilberger, Doron. *Notes from the course* Experimental Mathematics, *Rutgers University, Spring 2007 (taken by Humberto Montalván-Gámez).*

Zeilberger, Doron. *Notes from the course* Experimental Mathematics, *Rutgers University, Spring 2008 (taken by Humberto Montalván-Gámez).*

3.2 References for 1.2 Graph Theory

Diestel, Reinhard. *Graph theory.* Third edition.

Komlós, János. *Notes from the course* Applied Graph Theory, *Rutgers University, Spring 2008 (taken by Humberto Montalván-Gámez).*

3.3 References for 1.3 Probabilistic Methods

Alon, Noga; Spencer, Joel H. *The Probabilistic Method.* Second edition.

Durrett, Richard. *Probability: Theory and Examples.* Third edition.

Kahn, Jeffrey. *Notes from the course* Combinatorics I, *Rutgers University, Fall 2007 (taken by Humberto Montalván-Gámez).*

Vu, Van. *Notes from the course* Combinatorics II, *Rutgers University, Spring 2008 (taken by Humberto Montalván-Gámez).*

3.4 References for 2 Diophantine Approximations

Beck, József. *Notes from the course* Selected Topics in Discrete Math, *Rutgers University, Spring 2008 (taken by Humberto Montalván-Gámez).*

Iwaniec, Henryk. *Notes from the course* Selected Topics in Number Theory: Diophantine Approximations and Transcendental Number Theory, *Rutgers University, Spring 2009 (prepared by prof. Iwaniec).*

Schmidt, Wolfgang M. *Diophantine Approximations and Diophantine Equations.*

Steuding, Jörn. *Diophantine Analysis.*