Syllabus for Oral Examination
Haoyuan Xu

Functional Analysis

Banach spaces

(1) Linear Normed Spaces
(2) Basis and Barie Category Theorem
(3) Strong, Weak and Weak* Convergence
(4) Reflexibility of a Banach Space
(5) Dual Spaces
(6) Hahn-Banach Theorem
(7) Alaogulu Theorem

Hilbert Spaces

(1) Riezs Lemma
(2) Orthonormal sets and Bases
(3) Bessel's inequality and arseval's Theorem

Linear Operators

(1) Uniform Bounded Principle
(2) Open Mapping Theorem, Inverse Mapping Theorem and Closed Graph Theorem
(3) Fixed Points Theorem
(4) Riezs-Thorin Interpolation Theroem
(5) Compact Operators
- Resolvent Set
- Resolvent Operators
- Spectrum
(6) Spectrum of a Compact Operator
(7) Self-adjoint Operator

Partial Differential Equations

Laplacian Equation

(1) Mean-value Properties

(2) The Maximum Principle

(3) Harnack Inequality

(4) Perron's Method

(5) Green's Representation

(6) Fundamental Solution

(7) Newtonian Potential

Sobolev Spaces

(1) Extension Theorem

(2) Sobolev Embedding Theorem

(3) Sobolev Inequalities(Morrey,Gagliardo-Nirenberg-Sobolev Inequality)

(4) Poicaré's Inequality

(5) Compact Imbedding Theorem

Second Order Elliptic Equations

(1) Classical Maximum Principle

(2) Uniqueness of Dirichlet and Neumann Boundary Value Problems

(3) Existence theorems
- Method of Continuity
- Lax-Milgram Theorem
- Fredholm Alternative Theorem

(4) Regularity of Solutions and Schauder Interior and Global Estimates