Oral Exam Syllabus

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I. Combinatorics, Graph Theory, and the Probabilistic Method

Combinatorics

Basic Enumeration: counting arguments, generating functions, recurrence relations, reflection principle, inclusion-exclusion.

Set Systems: Erdös–Ko–Rado, Ray-Chaudhuri–Wilson, Frankl–Wilson, Sperner's Theorem, Littlewood–Offord, Kruskal-Katona, Harper's Theorem, Fisher's and Generalized Fisher's Inequality, Steiner systems, Baranyai's Theorem, Kahn-Kalai.

Lattices: Geometric and distributive lattices, Birkhoff covering property, Jordan-Dedekind chain condition, Incidence Algebra, Möbius inversion, Weisner's Theorem, Dowling-Wilson Theorem.

Correlation Inequalities: Harris, Holley, FKG, four funcitons, Shepp's theorem, stochastic domination, [1/3, 2/3]-conjecture, Strassen's theorem.

Ramsey Theory: Ramsey's Theorems, probabilistic lower bounds, Van der Waerden's Theorem, stepping-up lemma.

Discrepancy: Erdös-Selfridge Theorem, Beck-Fiala, Roth's theorem on 2D discrepancy, discrepancy in arithmetic progressions, linear and hereditary discrepancy, 6 standard deviations suffice, Komlós conjecture.

References: van Lint and Wilson, *A Course in Combinatorics*

Graph Theory

Algorithms: Minimum Weight Spanning Tree, Dijkstra, Hungarian Method, Edmonds Matching Algorithm, Kruskal.

Matching Theory: Hall, Stable Marriage, Tutte's 1-factor Theorem, Augmenting Paths.

Connectivity: Mincut-Maxflow, Menger's Theorem.

Coloring: Brooks' Theorem, Vizing's Theorem, König's (edge-coloring) Theorem, Hadwiger's conjecture, perfect graphs, weak perfect graph theorem, proof of the Dinitz Conjecture, 5-color theorem.

Extremal: Turán's Theorem, Erdös-Stone Theorem, Ramsey-type results (R(k, k) bounds), Regularity Lemma.

Planarity: Euler's formula, Kuratowski, Wagner.

References: Diestel, *Graph Theory*

<u>Probabilistic Method</u>

Basics: linearity of expectation, Bonferroni inequalities, binomial and Poisson distributions, conditional probabilities, law of total probability, Chebyshev's inequality, Chernoff bound, coupling and stochastic domination.

Second Moment Method: General procedure, threshold function for containg a given subgraph.

Lovász Local Lemma: summmetric and general versions, applications to linear arboricity conjecture and Latin transversals.

Martingales: Azuma's inequality, edge and vertex exposure, application to chromatic number.

Poisson paradigm: Janson inequalitites, application to number of triangles in $G_{n,p}$, Brun's sieve, application to number of isolated points in $G_{n,p}$.

Random graphs: monotone properties, $G_{n,p}$ vs. $G_{n,k}$, threshold functions, probabilistic refutation of Hajós conjecture, connectedness.

References: Alon and Spencer, *The Probabilistic Method*

II. Discrete Optimization

Basics: Linear Programming, Simplex and Dual Simplex methods, Duality Theorems, Farkas' lemma, Integer programming.

Min-Max theorems: Covering and packing problems, Fekete's lemma, Menger, Dilworth, König

Algorithms: P, NP, co-NP and good characterizataions, approximation results, knapsack problem.

Geometry: Convexity, cones, polyhedra and polytopes, polarity, blocking and anti-blocking polyhedra, integer polyhedra.

Integer programming: Totally Unimodular matrices, total dual integrality, Hilbert basis, integer hull, cutting planes.

References: Schrijver, *Combinatorial Optimization – Polyhedra and Efficiency*