

JAKE BARON'S ORAL QUAL SYLLABUS

COMMITTEE: JEFF KAHN (CHAIR), SIMON THOMAS, JÓZSEF BECK, DORON ZEILBERGER

1. COMBINATORICS

Enumeration: bijections, generating functions, binomial and multinomial coefficients, recurrence relations, inclusion-exclusion, Stirling's formula

Hypergraphs: Sperner, LYM inequality, Erdős-Ko-Rado, Kruskal-Katona, Fisher's Inequality (and generalized Fisher), Ray-Chaudhuri-Wilson, Frankl-Wilson, Harper, Baranyai

Posets and Lattices: Dilworth, linear extensions of posets, $\frac{1}{3}$ - $\frac{2}{3}$ conjecture, distributive and geometric lattices, Birkhoff representation theorem, Möbius inversion, Weisner, Dowling-Wilson

Correlation Inequalities: Harris-Kleitman, Fortuin-Kasteleyn-Ginibre, Ahlswede-Daykin, Shepp XYZ

Discrepancy: Beck-Fiala, Roth's $\frac{1}{4}$ -theorem on arithmetic progressions

Ramsey Theory: Ramsey, infinite Ramsey, König tree lemma, probabilistic lower bounds, van der Waerden, statement of Szemerédi

2. PROBABILISTIC METHODS

Basics: linearity of \mathbb{E} , \cup -bound and Bonferroni inequalities, Chebychev's inequality, Chernoff bounds, alteration methods

Second Moment Method: application to threshold function for containing a fixed subgraph

Local Lemma: symmetric and general versions, applications to hypergraph discrepancy, Ramsey lower bounds, Latin transversals

Poisson Paradigm: Janson's inequality and application to number of triangles in $G_{n,p}$, Brun's sieve and application to number of isolated vertices in $G_{n,p}$

Martingales: vertex and edge exposures, Azuma's inequality and application to chromatic number

Random Graphs: $G_{n,p}$ vs. $G_{n,M}$, monotone properties, existence of threshold functions, Bollobás-Thomason, probabilistic refutation of Hájos conjecture

Entropy: basic properties, Shearer's lemma, application to Minc Conjecture

Discrete Random Choice: Balog-Szemerédi-Gowers

3. GRAPH THEORY

Matchings: König, Hall, Tutte, stable matchings, matching polytopes

Connectivity: Kruskal's algorithm for minimum weight spanning tree, Menger, max-flow-min-cut, structure of 2-connected graphs

Planarity: Euler's formula, Kuratowski, Wagner

Coloring: 5-color theorem, Brooks, Vizing, Thomasson's 5-list-coloring of planar graphs, perfect graphs, Lovász's proof of weak perfect graph theorem, Galvin's proof of Dinitz conjecture

Extremal: Turán, statement of regularity lemma, Erdős-Stone, Chvátal-Rödl-Szemerédi-Trotter

4. FOUNDATIONS

Recursion Theory: recursive functions, Ackerman function, Turing machines, Church-Turing thesis (statement), computable and recursively enumerable sets, Turing degrees, jump operator, halting problem

Model Theory: Gödel completeness, Henkin theories, compactness, ultrafilters, ultraproducts, Łoś, types, omitting types, Löwenheim-Skolem

Basic Set Theory: ordinals, cardinals, cardinal arithmetic, König's lemma, equivalent forms of Choice, \diamond , MA, Aronszajn trees, Suslin trees, Root system lemma

Forcing: forcing theorems (statements), types of ultrafilters on ω (Ramsey, selective, weakly selective)

Descriptive Set Theory: Polish and standard Borel spaces, Borel isomorphism theorem, Borel and projective hierarchies