# ORAL QUALIFICATION EXAM SYLLABUS

JARET FLORES

## 1. Commutative Algebra

### 1.1. General Theory of Rings & Modules.

- Associated primes and primary decomposition
- Chain conditions, Noetherian and Artinian rings
- Integral dependence and valuations
- Noether normalization and Hilbert Nullstellensatz
- DVR's and Dedekind Domains
- Fractional ideals

### 1.2. Completions.

- Topologies, completions and Hensel's lemma
- Filtrations and Krull's Intersection Theorem
- Graded rings and modules
- The associated graded ring

### 1.3. Dimension Theory.

- Hilbert functions
- Dimension theory of Noetherian local rings
- Regular local rings

### 1.4. Local Cohomology.

- Injective modules
- Matlis duality
- Cohen-Macaulay and Gorenstein rings
- Vanishing theorems and structure of $H^d_m(R)$

## 2. Homological Algebra

### 2.1. Chain Complexes.

- Chain complexes, exactness
- Chain maps, chain homotopies, mapping cones and cylinders
- Double complexes, total complexes

### 2.2. Derived Functors.

- $\delta$-functors
- Projective and injective resolutions
- Left and right derived functors
- Adjoint functors, left/right exactness, Tor, Ext
- Balancing Tor and Ext

### 2.3. Tor and Ext.

- Tor for abelian groups
- Flatness, computing Tor from flat resolutions
- Relating Ext and extensions
- Derived functor of the inverse limit: $\lim^1$
- Künneth formula, Universal Coefficient theorem

### 2.4. Homological Dimension.

- Dimensions
- Rings of small dimension
- Change of rings theorems
- Local rings
- Koszul complexes
- Local cohomology

### 2.5. Sheaf Cohomology.

- Cohomology with support
- Vanishing theorem of Grothendieck

## References

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[3] Hartshorne, R. *Algebraic Geometry.* Springer, 1977.
[4] Huneke, C. "Lectures on Local Cohomology," 2004.
[5] Weibel, C. *An Introduction to Homological Algebra.* Cambridge U. Press, 1994.