ORAL QUAL SYLLABUS:
SET THEORY; RECURSION THEORY

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Set Theory.

- Basic set theory (Jech 1-8)
  - Königs lemma. Cardinal arithmetic in ZFC.
  - Ramsey's theorem.
  - Suslin's problem. There exists an Aronszajn tree.
  - $\diamondsuit$ implies there is a Suslin tree, MA implies there is not.
  - Root system lemma.
- Forcing basics (Kunen 7,8)
  - Fundamental forcing theorems (statements).
  - Force CH. Force $\neg$CH. Force $\diamondsuit$.
  - Chain and closure conditions on a partial order.
  - Cohen forcing.
  - Product forcing.
  - Consistency of MAC+$\neg$CH.
  - Consequences of MA, e.g. 2^ω is regular.
  - Easton's theorem.
- Descriptive set theory (Srivastava 1-4)

  Polish and Standard Borel Spaces
  - Basic definitions and examples.
  - The Borel Isomorphism Theorem.
  - Borel-generated topologies and the Ramsey-Mackey Theorem.
  - Sequential trees. Systems of sets and their associated maps. Souslin, Lusin, and Cantor schemes. The Souslin operation $\mathcal{A}$.

  The Borel and Projective Hierarchies.
  - Basic definitions and facts, including closure properties.
  - Existence of universal sets for each. Non-collapsing of each.
  - Every uncountable Polish space contains an analytic set that is not Borel.
  - The reduction theorem for additive Borel classes and the separation theorem for multiplicative Borel classes.
  - Equivalence of various definitions of analytic sets.
  - Every coanalytic set is a union of $\aleph_1$ Borel sets.
  - Definitions of Σ^1_1-complete, Π^1_1-complete. WF is Π^1_1-complete.
  - Regularity properties: every analytic subset of a Polish space is measurable, has the Baire property, and has the perfect set property.
  - Souslin's Theorem and the First Separation Theorem for analytic sets.

Recursion Theory.

- Basic definitions and theorems (Soare 1-4)
  - Recursive functions.
  - Unsolvable problems.
  - Recursively enumerable sets.
  - The Recursion Theorem.
  - Complete sets.
  - Relative computability.
  - Turing degrees.
  - The jump operator.
  - The arithmetical hierarchy.
- Constructions and methods (Soare 5-7)
  - Simple sets and Post's problem.
  - Oracle constructions of non-recursively-enumerable degrees.
    - Construct a pair of incomparable degrees below $\mathbf{0}'$
    - Cones of degrees
    - Inverting the jump operator
    - Upper and lower bounds for degrees
  - The finite injury priority method.

References.

- Jech, Thomas. *Set Theory* (Third Edition).
- Kunen, Kenneth. *Set Theory: An Introduction to Independence Proofs.*
- Srivastava, S.M. *A Course on Borel Sets.*
- Soare, Robert. *Recursively Enumerable Sets and Degrees.*