ORAL QUALIFYING EXAM

December 16, 2011

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Analytic Number Theory

- a) Analytic properties of the Riemann zeta function and Dirichlet L-functions.
- b) Dirichlet's theorem on primes in arithmetic progressions.
- c) Prime Number Theorem.
- d) Zero-free regions of Dirichlet L-functions and the exceptional zero problem.

Algebraic Number Theory

- a) Ring of integers in a number field: integral basis, discriminant, and different.
- b) Decomposition of primes.
- c) Class group and finiteness of class number; Minkowski's constant.
- d) Dirichlet's theorem on units.

Modular Forms

- a) Modular forms for the modular group and its congruence subgroups.
- b) Eisenstein and Poincare series.
- c) Structure of the ring of modular forms.
- d) Hecke operators.