

**Syllabus for Oral Qualifying Exam**  
James Dibble

**Major Topic: Differential and Riemannian Geometry**

Differential geometry

- Smooth manifolds
- Vector bundles
- Lie brackets and derivatives
- Submersions, immersions, and embeddings
- Integration and Stokes's Theorem
- de Rham cohomology
  - de Rham Theorem
  - Poincaré duality

Riemannian geometry

- Riemannian metrics
- First and second variation formulas
- Levi-Civita connection
- Geodesics and the exponential map
  - Gauss Lemma
  - Hopf-Rinow Theorem
- Jacobi fields
- Parallel transport
- Gauss-Bonnet Theorem
- Comparison geometry
  - Spaces of constant sectional curvature
  - Comparison estimates
  - Conjugate points
  - Cartan-Hadamard Theorem
  - Preissmann's Theorem
  - Bonnet-Myers Theorem
  - Synge's Theorem

References:

Lee, John. *Introduction to Smooth Manifolds*.  
Lee, John. *Riemannian Manifolds: An Introduction to Curvature*.  
Petersen, Peter. *Riemannian Geometry*.

## Minor Topic: Algebraic Topology

### Fundamental group

- Homotopy invariance
- Seifert-Van Kampen Theorem

### Covering spaces

- Lifting properties
- Deck transformations
- Classifications

### Singular homology and cohomology

- Degree of a map
- Euler characteristic
- Exact sequences and excision
- Mayer-Vietoris sequence

### References:

- Lee, John. *Introduction to Topological Manifolds*.
- Hatcher, Allen. *Algebraic Topology*.