

Oral Qual Syllabus

Jianguo Xiao

1 Primary Topic: Functional Analysis

1.1 Banach Spaces and Hilbert Spaces

1. Hahn-Banach theorems
2. Uniform boundedness principle
3. Open mapping theorem and the closed graph theorem
4. Dual spaces and reflexive spaces
5. Weak topology and weak* topology
6. Dual space of a Hilbert space
7. The Stone-Weierstrass theorem

1.2 Locally Convex Spaces

1. Seminorms and Minkowski functional
2. Fréchet spaces
3. Functions of rapid decrease and the tempered distributions
4. The N-representation for \mathcal{S} and \mathcal{S}'

1.3 Bounded Operators and Spectral Theorem

1. Self-adjoint, unitary, normal operators
2. Definition and basic properties of spectrum
3. Compact operators and Fredholm alternative
4. Spectral theorem

1.4 Unbounded Operators

1. Densely defined operators
2. Closed, symmetric, and examples
3. Spectral Theorem
4. Stone's Theorem
5. Convergence of unbounded operators

2 Secondary Topic: Probability Theory

2.1 Itô Integrals

1. Brownian motion
2. Construction of the Itô integral
3. Some properties of the Itô integral
4. Extensions of the Itô integral

2.2 The Itô Formula and the Martingal Representation

1. The one dimensional Itô formula
2. The multi dimensional Itô formula
3. The martingal representation theorem

2.3 Stochastic Differential Equations

1. Examples and some solution methods
2. An existence and uniqueness result
3. Weak and strong solutions

2.4 Diffusions: Basic Properties

1. The Markov property
2. The strong Markov property
3. The generator of an Itô diffusion
4. The Dynkin formula
5. The characteristic operator

3 References

1. Reed, M., Simon, B.: *Methods of Modern Mathematical Physics, Vol I: Functional Analysis*
2. Øksendal B.: *Stochastic Differential Equations*, Chapters 2-5 and Chapter 7.