

Proposed topics for the Ph. D. Qualifying Oral Examination
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Commutative Algebra

1. Basic Commutative Algebra

- 1.1 Localization of rings and modules
- 1.2 The Hilbert Nullstellensatz
- 1.3 Associated primes and primary decomposition
- 1.4 Integral dependence and valuations
- 1.5 DVRs, Dedekind rings, and Krull rings
- 1.6 Chain Conditions
- 1.7 Noetherian and Artinian rings
- 1.8 Completions

2. Hilbert Functions of Graded Modules

- 2.1 Graded rings and modules
- 2.2 Hilbert functions over homogeneous rings
- 2.3 Macaulay's theorem on Hilbert functions
- 2.4 Gotzmann's regularity and persistence theorem
- 2.5 Hilbert functions over graded rings
- 2.6 Filtered rings
- 2.7 The Hilbert-Samuel function and reduction ideals
- 2.8 Multiplicity

3. Cohen-Macaulay Rings

- 3.1 Regular sequences
- 3.2 Grade and depth
- 3.3 Depth and projective dimension
- 3.4 Cohen-Macaulay rings and modules
- 3.5 Regular rings and normal rings
- 3.6 Complete intersections

Homological Algebra and Gröbner Bases

1. Homological Algebra

- 1.1 Chain complexes
- 1.2 Chain homotopies
- 1.3 Mapping cones and cylinders
- 1.4 Projective and injective resolutions
- 1.5 Left and right derived functors
- 1.6 Adjoint functors and left/right exactness
- 1.7 Tor and Ext
- 1.8 Dimensions
- 1.9 Rings of small dimensions
- 1.10 Change of ring theorems
- 1.11 Koszul complexes
- 1.12 Local cohomology

2. Gröbner Bases

- 2.1 Orderings on the monomials in $k[x_1, \dots, x_n]$
- 2.2 Division algorithm in $k[x_1, \dots, x_n]$
- 2.3 Monomial ideals and Dickson's lemma
- 2.4 Hilbert basis theorem and Gröbner bases
- 2.5 Properties of Gröbner bases
- 2.6 Buchberger Algorithm
- 2.7 Applications of Gröbner bases

References

- [1] M.F. Atiyah and I.G. Macdonald, *Introduction to Commutative Algebra*, Addison-Wesley, 1969
- [2] W. Bruns and J. Herzog, *Cohen-Macaulay Rings*, Cambridge University Press, 1993
- [3] H. Matsumura, *Commutative Ring Theory*, Cambridge University Press, 1986
- [4] W.W. Adams and P. Loustau, *An Introduction to Gröbner Bases*, AMS, 1994
- [5] D. Cox, J. Little, and D. O'Shea, *Ideals, Varieties and Algorithms*, Springer-Verlag, 2nd edition, 1997
- [6] J.J. Rotman, *An Introduction to Homological Algebra*, Academic Press, 1979
- [7] W.V. Vasconcelos, *Computational Methods in Commutative Algebra and Algebraic Geometry*, Springer-Verlag, 1998
- [8] C.A. Weibel, *An Introduction to Homological Algebra*, Cambridge University Press, 1994