Syllabus for Oral Examination

Qinian Jin

Elliptic Partial Differential Equations

- **Laplace and Poisson Equations**

  (a) Mean value properties
  (b) The maximum principle
  (c) Harnack Inequality and Liouville's Theorem
  (d) Fundamental solution of Laplace equation
  (e) Green's representation formula and Poisson integral formula
  (f) Analyticity of harmonic functions
  (g) Perron method
  (h) Newtonian potential

- **Classical Solutions of Second Order Elliptic Equations**

  (a) Hopf lemma, Weak/strong maximum principle
  (b) Schauder interior and global estimates
  (c) Existence results by the method of continuity and Fredholm alternative
  (d) Interior and boundary regularity results

- **Sobolev Spaces**

  (a) Definition of Sobolev spaces
  (b) Meyers-Serrin theorem
  (c) Extension theorem
  (d) Gagliardo-Nirenberg-Sobolev inequality
  (e) Morrey inequality
  (f) Rellich-Kondrachov compact imbedding theorem
  (g) Poincare inequality
  (h) Difference quotients

- **Weak Solutions of Second Order Elliptic Equations**

  (a) The definition of weak solutions

(b) Existence results by Lax-Milgram theorem and Fredholm alternative
(c) Regularity of weak solutions
(d) Moser iteration, Hanarck inequality
(e) De Giorgi-Nash-Moser theorem

Riemannian Geometry

- Riemannian metrics
- Levi-Civita connection, Parallel translation
- Geodesics, exponential map, Gauss Lemma
- Normal neighborhood, Convex neighborhood
- Normal coordinates
- Hopf-Rinow theorem
- Curvature tensor
- Sectional curvature, Ricci curvature, Scalar curvature
- Jacobi fields, Conjugate points
- First and second variations of arc length
- Bonnet-Myers theorem
- Cartan-Hadamard theorem
- Index inequality—minimality of Jacobi fields
- Rauch comparison theorem
- Space forms
- Morse index theorem
- Differential operators: grad, div, ? and Hess; Divergence theorem
- Riemannian submanifolds
  (a) The second fundamental forms
  (b) Weingarten equation, Gauss equation, and Codazzi equation
  (c) Submanifolds with codimension 1, Gauss curvature and mean curvature
  (d) Theorema egregium of Gauss
  (e) Gauss-Bonnet theorem

References

[1] Gilbarg, D and Trudinger, N. S, Elliptic Partial Differential Equations of Second Order, Springer, 1983.

[2] Evans, L.C. Partial Differential Equations. AMS Providence, 1998

[3] Jost, J, Partial Differential Equations, GTM 214, Springer, 2002.

[4] Lee, John M, Riemannian Manifolds, Springer, GTM 176, 1997.

[5] Carmo, M. P. do, Riemannian Geometry, Birkhauser, Boston, 1992.