

Oral Qualifying Exam Syllabus

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1 Major topic: Math Finance

1.1 Brownian Motion

- Scaled random walk
- Brownian motion: definition, filtration, martingale property
- First-order variation, quadratic variation
- Reflection principle

1.2 Stochastic Calculus

- Ito's integral: construction, properties
- Ito formula
- Black-Scholes-Merton equation
- Put-call parity

1.3 Risk-Neutral Pricing

- Girsanov's theorem
- Risk neutral measure, pricing under the risk-neutral measure
- Martingale representation theorem
- Fundamental theorem of asset pricing

1.4 Connections with Partial Differential Equations

- The Markov property
- Feynman-Kac theorems
- Derivation of the Black-Scholes PDE

1.5 Exotic Options

- Knock-out barrier options
- Lookback option
- Asian option

1.6 Volatility Models

- Implied volatility, volatility smile
- Local volatility
- Relating implied volatility with local volatility
- Heston model

1.7 Jump Processes

- Poisson process, Poisson random measure
- Compound Poisson process, jump measure of compound Poisson process
- Levy process, Levy measure
- Levy-Ito decomposition
- Levy-Khinchin representation

2 Minor topic: Partial Differential Equations

2.1 Heat Equation

- Derivation of the fundamental solution
- Maximum principle, uniqueness
- Energy methods

2.2 Second Order Elliptic Equations

- Definition of weak solution
- Existence of weak solutions via Lax-Milgram theorem

- Weak maximum principle
- Hopf Lemma
- Strong maximum principle

2.3 Second Order Parabolic Equations

- Definition of the weak solutions
- Existence of weak solutions, Galerkin approximations
- Weak maximum principle
- Harnack's inequality
- Strong maximum principle

References

- [1] Steven E. Shreve *Stochastic Calculus for Finance II*, Springer, 2003.
- [2] Jim Gatheral *Case Studies in Financial Modelling course notes*, Courant Institute, 2005
- [3] Rama Cont *Financial Modelling with Jump Processes*, Chapman & Hall/CRC, 2003
- [4] L.C. Evans, *Partial Differential Equations*, AMS Providence, 1998.