Oral Exam
Combinatorics, Graph Theory, Theory of Boolean Functions

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1 Combinatorics

Basic Enumeration counting arguments, generating functions, recurrence relations, inclusion-exclusion

Set Systems: Sperner's theorem, Erdos-Ko-Rado, Kruskal-Katona, Fisher's Inequality, Frankl Wilson

Correlation Inequalities Kleitman's Lemma, FKG inequality, four function theorem

Ramsey Theory Ramsey's Theorem, infinite Ramsey theory, probabilistic lower bounds, var der Waerden

Discrepancy 6 Standard Deviations Suffice, Beck-Fiala

2 Graph Theory

Matching Hall's Theorem, Konig's Theorem

Connectivity Menger's Theorem, Max-Flow Min-Cut, Kruskal's Algorithm

Coloring: Brook's Theorem, Vizing's Theorem, 5-color theorem

Extremal Problems Turan's Theorem, Statement of Regularity lemma, Erdos-Stone Theorem, Ramsey bounds, $ex(n, P_k)$ (proof contains Dirac's Theorem), $ex(n, C_4)$, $ex(n, K_{r,s})$

Random Graphs: Number of triangles, clique number, threshold for connectedness

Planar Graphs Euler Characteristic, proof that $K_5, K_{3,3}$ are not planar, Kuratowski's Theorem, Crossing number.

3 Probabilistic Methods

Basics Markov Inequality, Chebyshev's Inequality, Chernoff bound, binomial and Poisson distributions, law of total probability

Alteration Method property B, probabilistic construction of high girth and high chromatic number graph

Second Moment Method: application to threshold functions for $G_{n,p}$

Lovasz Local Lemma: Symmetric and general versions, application to Ramsey Bounds, $R(3,k)$

Poisson Paradigm: Janson inequalities, number of triangles in $G_{n,p}$, number of isolated points

4 Boolean Functions

Basic Examples: MAJ_n, TRIBES, addressing function

Representations: Formula, Polynomial, Circuit, Decision Tree, Branching Program

Measures of Complexity: Decision Tree, Certificate Complexity, Polynomial degree, sensitivity, block sensitivity, DNF, CNF size, bounds relating complexities

Changing between Representations: Barrington's Theorem (group program), $bs(f) \leq C(f) \leq D(f)$, $D(f) \leq C^2$, $bs(f) \leq 2deg(f)^2$, $D(f) \leq deg(f)^2 bs(f)$

Randomization Lower Bounds: $R(f) \geq \sqrt{D(f)}$, $R(P) \geq t$ for (t,t) bipartite graphs, $R(f) \geq \left(\frac{n}{2\sqrt{p(1-p)}}\right)^{\frac{2}{3}}$ for f monotone + weakly symmetric (p the critical probability), Yao's theorem for $R(P)$ for monotone bipartite graph properties.

Testing and decoding: BLR and Hastad Tests, Dictatorship testing

Influence of variables: Uses of Fourier analysis, $Inf_i(f) = \sum_{J:i\in J} \hat{f}(J)^2$, Friedgut's Theorem, Kahn Kalai Linial