James Taylor's Oral Exam Syllabus

I. Functional Analysis
- A. Basics of Banach spaces
  - i. Examples such as L^p spaces, sequences spaces, direct sums, and quotients
  - ii. Linear functionals: duals, reflexive spaces, and the Hahn Banach theorem
  - iii. Baire category, open maps, closed graphs, and Banach-Steinhaus
  - iv. Basics of Hilbert spaces (polarization, adjoints, Riesz lemma)
- B. Useful topological notions
  - i. Nets
  - ii. Compactness (Tychonoff's Theorem, Urysohn's Lemma, Stone-Weierstrass Theorem)
  - iii. Measure theory on compact spaces (Riesz-Markov Theorem)
  - iv. Various topologies on operator spaces
  - v. The Banach-Alaoglu Theorem
- C. Bounded Operator Theory
  - i. Adjoints
  - ii. Spectrum
  - iii. Positive operators, square roots
  - iv. Compact operators
  - v. Fredholm operators and the Fredholm Alternative
  - vi. Spectral Mapping Theorem
  - vii. Functional Calculus
  - viii. Various Spectral Theorems and measures related to self-adjoint operators
- D. Unbounded Operator Theory
  - i. Definitions and generalizations
  - ii. Self-adjoint, closed, essentially self-adjoint, and symmetric operators
  - iii. Cayley Transform
  - iv. Spectral Theorem for unbounded self-adjoint operators
  - v. Stone's Theorem
- E. Differential Calculus on Banach Spaces
  - i. Derivative of operators on Banach spaces and generalizations of elementary differential calculus
  - ii. Inverse and Implicit Function Theorems
  - iii. Infinite Dimensional Manifolds and their issues
- F. Differential Operators and Spectral Theory
  - i. Schwarz space
  - ii. Fourier Transform
  - iii. Distributions
  - iv. Sobolev Spaces
  - v. Various Laplace operators and their uses
- G. Applications to Quantum Mechanics
  - i. Basics of Bohmian Mechanics and the formalism of Quantum Mechanics
  - ii. Position, momentum operators
  - iii. One-dimensional problems such as the harmonic osciallator, different potential wells
  - iv. Spin
  - v. Bell's Theorem

II. Differential Geometry
  A. Basic Definitions and examples
    i. Definitions of Manifolds, tangent vectors, vector fields, vector bundles
    ii. Examples: Surfaces, Lie groups-Matrix groups, submanifolds
    iii. Various mappings such as immersions, induced maps
    iv. Quotient manifolds: Projective spaces, Grassmann manifolds
  B. Tensors and differential forms
    i. Tensors of all types, tensor fields, maps and tensors
    ii. Exterior algebra, exterior derivative, differential forms
    iii. Orientability and n-forms
    iv. Symmetrizing, alternating, contracting, and multiplying tensors
    v. Tensor Derivations
    vi. Lie Derivatives
    vii. Poincare Lemma and its partial converse
  C. Vector fields
    i. Existence and Uniqueness Theorems for ODE
    ii. One-Parameter groups
    iii. Vector Fields as flows and as differential operators
    iii. Lie algebra of vector fields
    iv. Frobenius' Theorem and foliations
  D. Metrics and Connections
    i. Definition of Metrics and Connections
    ii. Covariant Derivative
    iii. The Levi-Civita connection
    iv. Parallel Translation
    v. Geodesics
    vi. Frame fields
    vii. Exponential Map
    viii. Hopf-Rinow Theorem
    ix. Curvature
  E. Integration on Manifolds
    i. Definition of the integral
    ii. Manifolds with boundary
    iii. Stokes' Theorem
  F. Surface theory
    i. Fundamental Forms, Gauss Curvature, Principal Curvature
    ii. The Gauss Theorem
    iii. Gauss-Bonnet Theorem