Oral Qualifying Exam Syllabus

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Committee: Haïm Brezis, Eric Carlen, Sagun Chanillo, Zheng-Chao Han

Functional Analysis and C^* Algebras

1. Banach Spaces
   (a) Weak Topology
   (b) Weak* Topology
   (c) Reflexive Spaces
   (d) Separable Spaces
2. Hilbert Spaces
   (a) Projection Lemma
   (b) Riesz Representation Theorem
   (c) Stampacchia and Lax-Milgram
   (d) Hilbert Sums and Orthonormal Bases
3. Banach Algebras
   (a) Definitions
   (b) Spectral Theory of Banach Algebras
   (c) The Maximal Ideal Space and the Gelfand Transform
4. C^* Algebras
   (a) Involutions and C^* Algebras
   (b) The Commutative Gelfand-Naimark Theorem
   (c) The Abstract Spectral Theorem
   (d) The Spectral Mapping Theorem
   (e) Positivity in C^*-algebras
5. Bounded Operators on a Hilbert Space
   (a) Definitions
   (b) The Spectral Theorem for Normal Operators on a Hilbert Space
   (c) The Measurable Function Calculus
6. Compact Operators on a Hilbert Space
   (a) Compact Operators as the Closure of Finite Rank Operators
   (b) Fredholm Alternative
   (c) Riesz-Schauder Theorem (Spectrum of a Compact Operator)
   (d) Spectral Decomposition of Self-Adjoint Compact Operators

Partial Differential Equations

1. Laplace Equation
   (a) Fundamental Solution
   (b) Properties of Harmonic Functions: Mean-Value Property, Maximum Principles, Regularity, Liouville's Theorem, Harnack's Inequality
   (c) Green's Functions on the Half-Space and Ball
   (d) Energy Methods: Uniqueness and Dirichlet's Principle
2. Heat Equation
   (a) Fundamental Solution
   (b) Maximum Principle, Uniqueness, Regularity
   (c) Energy Methods: Uniqueness and Backward Uniqueness
3. Sobolev Spaces
   (a) Weak Derivatives
   (b) Sobolev Spaces
   (c) Approximation by Smooth Functions
   (d) Difference Quotients
   (e) The Space $W_0^{1,p}$.
4. Sobolev Inequalities
   (a) n=1
   (b) Sobolev Inequalities, $1 \leq p < n$ (GNS, Poincaré)
   (c) Sobolev Inequalities, $p = n$
   (d) Sobolev Inequalities, $n < p \leq \infty$ (Morrey)
   (e) Rellich-Kondrachov Compactness Theorem
5. Second-Order Elliptic Equations
   (a) Definition of Elliptic Equations and Weak Solutions
   (b) Existence of Weak Solutions via Energy Estimates
   (c) Existence of Weak Solutions via Fredholm Alternative
   (d) Interior Regularity
   (e) Weak Maximum Principle
   (f) Eigenvalues of Symmetric Elliptic Operators Form a Basis for L^2