Ph.D. Oral Qualifying Exam Syllabus

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1 Combinatorics

1.1 Enumerative Combinatorics

- Enumeration, pigeonhole principle
- Generating functions, recurrences, binomial coefficients, Catalan numbers
- Stirling numbers, Stirling's formula, connection coefficients
- Partial orders, incidence algebra, principle of inclusion/exclusion, Möbius inversion
- Lattices, Birkhoff representation theorem, & occurrence in abstract algebra
- Integer partitions, Ferrer's diagrams, basic identities

References: Stanley - *Enumerative Combinatorics*, van Lint & Wilson - *A Course in Combinatorics*, course notes MATH 642:582-3, Andrews & Eriksson - *Integer Partitions*

1.2 Graph Theory

- Kőnig's theorem, Hall's Marriage theorem, Menger's theorem
- Kuratowski's & Wagner's theorems, Euler's formula, 5-color theorem
- Turán's theorem, statement of Erdős-Stone theorem
- Vizing's theorem, Brook's theorem
- Statements of Seymour-Robertson theorem, Hadwiger's conjecture
- Statement of Szemerédi's regularity lemma
- Cayley graphs & graph automorphisms
- Random graphs ($G_{n,p}$), the countable universal graph

References: Bollobás - *Modern Graph Theory*, Diestel - *Graph Theory*, course notes MATH 642:581

1.3 Experimental & Computational Methods

- Symbolic computation, Maple & Mathematica
- Methods & applications: backtracking, simulated annealing, memoization
- Applications to Ramsey theory

1.4 Ramsey Theory

- Kőnig's lemma, Ramsey's Theorem - finite/infinite, graph/hypergraph
- Compactness theorem for hypergraphs, derivation from first-order compactness
- Probabilistic lower bounds and classical upper bounds
- Van der Waerden's theorem & generalizations, statement of Szemerédi's theorem
- Schur's theorem and Rado's theorem
- Hadwiger-Nelson problem, known upper & lower bounds

References: Graham, Rothschild, & Spencer - *Ramsey Theory*, Landman & Robertson - *Ramsey Theory on the Integers*

2 Analytic and Additive Methods & Results

- Elementary Fourier analysis & character theory on finite, discrete abelian, or classical (i.e. $\mathbb{R}$) groups
- Cauchy-Schwarz, Parseval, Plancharel, & Hausdorff-Young inequalities
- Additive energy & Ruzsa distance
- Balog-Szemerédi-Gowers theorem
- Plünnecke inequalities & Ruzsa's theorem
- Generalized arithmetic progressions (definitions & relation to sumsets)
- Freiman's theorem & Freiman homomorphisms
- Meshulam's theorem & finite field models
- Bohr sets, regular Bohr sets, definitions, properties, & relation to GAPs
- Roth's theorem & the circle method
- Gowers norms and $\mathcal{U}_2, \mathcal{U}_3$ inverse theorems

References: Tao & Vu - *Additive Combinatorics*, Rudin - *Real and Complex Analysis*, Rudin - *Fourier Analysis on Groups*, Nathanson -*Additive Number Theory*, course notes MATH 642:592, 642:588, 640:556