Syllabus for Oral Qualifying Exam

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Major Topic: Partial Differential Equations

• **Laplacian Equation**
(1) Fundamental Solution.
(2) Mean-value formulas and Converse to Mean-value property.
(3) Properties of harmonic functions:
Maximum principles, Smoothness, Local estimates, Liouville's theorem, Harnack's inequality, removable singularity, Schwarz reflection principle.
(4) Green's functions for a ball and for a half-space.
(5) The classic Dirichlet problem by Perron's method.
(6) The Dirichlet's principle by variational method.

• **Heat Equation**
(1) Fundamental Solution.
(2) Maximum principles on bounded $\overline{U}_T$ and $R^n \times [0, T]$.
(3) Backward uniqueness by energy methods.

• **Wave Equation**
(1) d'Alembert's formula, Kirchhoff's formula.
(2) Uniqueness and domain of dependence by energy methods.

• **Sobolev Space**
(1) Definition of Sobolev Space.
(2) Approximation by smooth functions.
(3) Extensions, Traces.
(4) Gagliardo-Nirenberg-Sobolev inequality, Morrey's inequality.
(5) Rellich-Kondrachov compact imbedding theorem.
(6) Additional Topics: Poincare's inequality, Difference quotients.

• **Second-Order Elliptic Equations**

(1) The definition of weak solutions.

(2) Existence by Lax-Milgram theorem and energy estimates, by Fredholm alternative.

(3) Interior regularity and boundary regularity.

(4) Hopf's lemma, Weak/Strong Maximum principles, Hanarck's inequality.

(5) Eigenvalues and eigenfunctions of symmetric elliptic $Lu = -\sum_{i,j=1}^{n}(a^{ij}u_{x_i})_{x_j}$

• **Second-Order Parabolic Equations**

(1) The definition of weak solutions.

(2) Existence by Galerkin approximations and Energy estimates; Uniqueness.

Minor Topic: Functional Analysis

• **Banach Spaces**

(1) Metric spaces, Contraction Mapping Principle.

(2) Normed vector spaces, Linear functionals: Hahn-Banach Theorem.

(3) Barie Category Theorem, Open and Inverse Mapping Theorem, Closed Graph Theorem, Uniform Bounded Principle.

(4) Dual and Reflexive spaces; Strong, Weak and Weak* Convergence.

• **Hilbert Spaces**

(1) Riesz Representation Theorem, Lax-Milgram Theorem.

(2) Orthonormal sets, Bases, Bessel's inequality, and Parseval's theorem.

(3) Orthogonal projection.

• **Compact(linear) operator**

(1) Definition.

(2) Fredholm alternative.

(3) Spectrum of a compact operator.

(4) Hilbert-Schmidt theorem.

Reference:

Lawrence C. Evans: *Partial Differential Equations*

Gilbarg-Trudinger: *Elliptic Partial Differential Equations of Second Order*

Peter Lax: *Functional Analysis*