

1 Combinatorics I & II

Basic Enumeration

- Stirling numbers: $1^{st}, 2^{nd}$, unsigned
- Representations of Permutations
- Generating Functions

Principle of Inclusion–Exclusion

Partially Ordered Sets

- Dilworth’s p.o. on {antichains}
- Dilworth’s Thm
- Sperner’s Thm

Lattices

- Distributive
 - Fund Thm Finite Distrib Lattices
- Geometric
 - Birkhoff Covering Property
 - Jordan–Dedekind Chain Condition

Closures

- Convex, Linear, Graphic, Ideal

Möbius Functions

- incidence algebra $I(P), \zeta, \mu$
- Möbius Inversion

- Weisner’s Thm
- Dowling–Wilson Thm

Hypergraphs

- Fisher’s & Generalized Fisher’s Inequality
- Nonuniform: Modular RC–W Thm

Extremal Problems for Finite Sets

- Intersecting Families
- Erdős–Ko–Rado Thm
- Shadows
 - Kruskal–Katona Thm
 - * Numbers in KK
 - Lex Order, Upper Shadow
- Turán’s Thm
 - Turán–type Problem

Correlation Inequalities

- Positively Correlated, Percolation
- Harris/Kleitman Thm
- Chebyshev’s Inequality
- FKG Inequality
- Ahlswede–Daykin “Four Fcns Thm”
 - Applications to Posets

Ramsey Theory

- Ramsey on Graphs

- Existence
- Erdős Bound $R(k, k) > \frac{k2^{k/2}}{e\sqrt{2}}$
- General Ramsey (on Hypergraphs)
- Infinite Ramsey

2 Graph Theory

Minimum Weight Spanning Tree

Matching Theory

- Bipartite
 - König’s Thm
 - Hungarian Method
 - Hall’s Thm
 - SDRs
 - Latin Rectangles
 - * Gale–Ryser Thm
- Baranyai’s Theorem
- Tutte’s 1–factor Thm
- Berge’s Thm

Max Flow Min Cut Theorem

Menger’s Thm

Connectivity

Vertex Colorings

- Brook’s Thm

Edge Colorings

- Vizing’s Thm

Planar Graphs

- Euler’s Formula
 - 5–color Thm (Bondy–Murty Proof)
- Kuratowski’s Theorem

Matroids

- Independence, Circuit, Rank, Base and Closure Axioms
- Linear, Graphic, Cographic
- Union of Matroids
- Duality
 - Abstract Duals
 - Whitney’s “Abstract Dual \iff Planar” Thm
- Edmond’s Intersection Thm
- Matroid Union Thm

Extremal Graph Theory

- * Turán ... (Comb)
- Erdős–Stone Thm
- Szemerédi’s Regularity Lemma

3 Probabilistic Methods

Chapters 1–6 and 8 in Alon–Spencer–Erdős