Syllabus – Kai Medville

1. Functional Analysis
   (a) Hilbert spaces
      i. Riesz lemma
      ii. orthonormal basis
   (b) Banach spaces
      i. Hahn-Banach theorem
      ii. Baire category theorem, principle of uniform boundedness, open mapping theorem, closed graph theorem
   (c) weak topologies, weak-$*$ topology, Banach-Alaoglu theorem
   (d) Fredholm alternative and the spectral theory of compact operators
2. Sobolev Spaces
   (a) density of smooth functions in Sobolev space
   (b) global extensions
   (c) trace theorem
   (d) Sobolev inequalities (Morrey, Gagliardo-Nirenberg-Sobolev)
   (e) compact imbedding
   (f) Poincare inequality
3. Laplace's Equation
   (a) the fundamental solution
   (b) Poisson's equation, $\Delta u = f$ in Ω, Ω a bounded domain, f Hölder continuous
   (c) mean value formula and maximum principle for subharmonic functions
   (d) Harnack's inequality
   (e) Green's function: for ball and half-space
   (f) single and double layer potentials
4. Second Order Elliptic Equations
   (a) strong and weak maximum principle
   (b) uniqueness of Dirichlet and Neumann boundary value problems
   (c) definition of weak solutions
   (d) existence: Lax-Milgram, energy estimates, Fredholm alternative
   (e) regularity of solutions
      i. interior and boundary regularity, smooth coefficients

ii. Hölder continuity for bounded coefficients

References

1. Partial Differential Equations by L.C.Evans
2. Elliptic Partial Differential Equationa of Second Order by Gilbarg and Trudinger
3. Maximum Principle by Protter and Weinberger
4. Functional Analysis I by Reed and Simon
5. Lectures on Partial Differential Equations by G.B. Folland