ORAL QUALIFICATION EXAM SYLLABUS

KNIGHT FU

1. Algebraic Geometry

1.1. Basic Concepts.

- Affine Varieties, Hilbert's Nullstellensatz
- Projective Varieties
- Morphisms of Varieties
- Rational Maps
- Nonsingular Varieties
- Intersection Theory, Bézout's Theorem

1.2. Sheaves and Schemes.

- Presheaves, Sheaves
- Schemes, Affine Schemes, Proj
- Separated and Proper Morphisms
- Sheaves of Modules, Quasi-Coherent Sheaves, Coherent Sheaves
- Weil Divisors, Cartier Divisors, Invertible Sheaves
- Projective Morphisms
- Differentials

1.3. Cohomology.

- Sheaf Cohomology, Hypercohomology
- Sheaf Cohomology of Noetherian Affine Schemes
- Čech Cohomolgy
- Cohomology of Projective Space
- Ext Groups and Sheaves

2. Homological Algebra

2.1. Chain Complexes.

- Chain Complexes, Exactness
- Chain Maps, Chain Homotopies, Mapping Cone and Cylinders
- Double Complexes, Total Complexes

2.2. **Derived Functors.**

- δ-functors
- Projective and Injective Resolutions
- Left and Right Derived Functors
- Adjoint Functors, Left/Right Exactness, Tor, Ext
- Balancing Tor and Ext

2.3. **Tor and Ext.**

- Tor for Abelian Groups
- Flatness, Computing Tor from Flat Resolution
- Relating Ext and Extensions
- Derived Functors of Inverse Limit: $\lim^1$
- Künneth Formula, Universal Coefficient Theorem

2.4. **Spectral Sequences.**

- Definition of Spectral Sequence, Convergence
- Leray-Serre Spectral Sequence
- Spectral Sequence of a Filtration
- Convergence Theorems
- Spectral Sequence of a Double Complex
- Cartan-Eilenberg Resolutions, Hyperderived Functors
- Grothendieck Spectral Sequences

2.5. **Derived Category.**

- Chain Complexes modulo Homotopy Equivalence
- Triangulated Category
- Localization, Gabriel-Zisman Theorem
- Derived Category
- Total Tensor Products
- Total Derived Functors of $\otimes_R S$
- Ext and $\mathbf{R}$Hom

References

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[2] Hartshorne, R., (1977). *Algebraic Geometry.* Springer.
[3] Shafarevich, I. R., (1977). *Basic Algebraic Geometry I and II.* Springer.
[4] Weibel, C., (1994). *An Introduction to Homological Algebra.* Cambridge U. Press.